Mathematics: applications and interpretation
Higher level
Paper 3

14 November 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 15 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

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Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 26]

This question considers whether it is reasonable to go on all the rides in a theme park and get back to the entrance in two and a half hours.

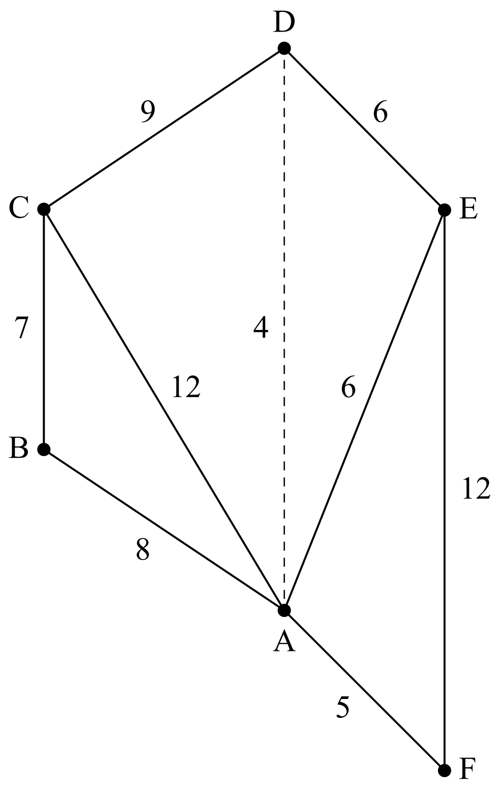
Martin is visiting a theme park. He will enter the park at $09{:}00$ and must leave the park by $11{:}30$. He uses information available on the internet to calculate whether he will be able to go on all of the rides in the two and a half hours.

He begins by constructing a graph which shows the main paths between the rides and the route of the cable car between the entrance/exit A and ride D.

His graph and the names of the rides are shown in the following diagram.

diagram not to scale

A	Entrance/Exit
B	Bullet
C	Canyon
D	Deep Dive
E	Energy Pulse
F	Flash



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(Question 1 continued)

The weights on the edges of the graph represent the times, in minutes, to walk between the rides and the time to travel by cable car between A and D.

Let T be the shortest possible time, in minutes, taken to visit all the rides, beginning and ending at A.

Martin notices that the graph contains a Hamiltonian cycle. He decides to use the weight of the Hamiltonian cycle as an upper bound for T.

(a) Find the weight of this Hamiltonian cycle. [2]

Martin constructs **Table 1** to show the shortest possible time it takes to travel between any two rides and between the entrance and any ride.

Table 1

	A	B	C	D	E	F
A	0	8	12	4	6	5
B	8	0	7	a	14	13
C	12	7	0	9	15	b
D	4	a	9	0	6	9
E	6	14	15	6	0	11
F	5	13	b	9	11	0

(b) Write down the value of

(i) a; [1]

(ii) b. [1]

(c) Use the nearest neighbour algorithm on **Table 1** to find an upper bound for T. [3]

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(Question 1 continued)

Martin decides to use the deleted vertex algorithm to find a lower bound for T by first deleting vertex A. The shortest possible time to travel between each ride, with vertex A deleted, is given in **Table 2**.

Table 2

	B	C	D	E	F
B	0	7	a	14	13
C	7	0	9	15	b
D	a	9	0	6	9
E	14	15	6	0	11
F	13	b	9	11	0

(d) (i) Use Prim's algorithm on **Table 2** to find the weight of the minimum spanning tree for the graph with vertices B, C, D, E and F.
Start at vertex B and write down the order in which the edges are selected. [3]

(ii) Hence find a lower bound for T. [2]

Martin finds more lower bounds for T, by deleting each vertex in turn. The results are shown in the following table.

Deleted vertex	**Lower bound**
B	39
C	39
D	36
E	36
F	39

Martin finds the smallest possible interval within which T lies, based on his calculated values for upper and lower bounds. He writes his answer in the form $p \le T \le q$.

(e) Write down the value of

(i) p; [1]

(ii) q. [1]

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(Question 1 continued)

Martin's favourite ride is Energy Pulse (E), so he decides to go there first. He plans to begin at A and visit the rides in the order E, D, C, B, F before returning to A. For the rest of the question, assume that Martin is taking this route.

(f) (i) Find the shortest possible time it would take to complete this route. [1]

(ii) State the edge which would need to be repeated. [1]

Each of the rides takes 2 minutes to complete.

Let the time spent waiting in the queue for ride B be written as the random variable B_t and similarly for the other rides.

The following distributions model the times spent waiting in the queues. Each waiting time is independent of all other waiting times and the time of day.

$$B_t \sim \mathrm{N}(13, 3), C_t \sim \mathrm{N}(21, 15), D_t \sim \mathrm{N}(16, 8), E_t \sim \mathrm{N}(10, 6), F_t \sim \mathrm{N}(20, 15)$$

(g) (i) Find the distribution for the total time spent queuing for all five rides. [3]

(ii) Find the probability that Martin manages to go on all five rides and return to the entrance in two and a half hours. [4]

Martin enters the park at $09{:}00$ and decides to follow his planned route, but has two consecutive rides on Energy Pulse.

(h) Find the expected time he will leave the park. [3]

2. [Maximum mark: 29]

In this question researchers are trying to find the most accurate model to use when modelling a population of wolves.

Historically, a population of wolves in an area had a stable size of 200. After some years of disruption, the population was reduced to 40 wolves. At this point, the area became a protected space and the population began to grow again.

Researchers in the area wish to model the size of the wolf population, x, as a function of t, where t is the time, in years, since the area became protected.

(a) Initially, the researchers consider using the logistic model

$$x = \frac{L}{1 + Ce^{-kt}} \text{, where } L, C, k \in \mathbb{R}^+.$$

The researchers decide to let $L = 200$.

(i) State the assumption being made in assuming $L = 200$. [1]

At $t = 0$, the population of wolves is 40.

(ii) Find the value of C. [2]

At $t = 5$, the population of wolves is found to have increased to 70.

(iii) Find the value of k. [2]

(iv) Use your model to predict the size of the wolf population in the area 10 years after it became protected. Give your answer correct to the nearest whole number. [2]

(b) An alternative model for population growth is called the Gompertz model. When applied by the researchers to the wolf population, this model satisfies the differential equation

$$\frac{dx}{dt} = ax\ln\left(\frac{200}{x}\right),\ a \in \mathbb{R}^+.$$

(i) Write down the value of $\frac{dx}{dt}$ when $x = 200$. [1]

(ii) Interpret your answer to part (b)(i) in context. [1]

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(Question 2 continued)

Consider the function $f(x) = \ln(\ln 200 - \ln x)$, where $0 < x < 200$.

(iii) Show that $f'(x) = \dfrac{-1}{x\ln\left(\dfrac{200}{x}\right)}$. [2]

(iv) Hence, use separation of variables to show that the general solution of

$$\frac{dx}{dt} = ax\ln\left(\frac{200}{x}\right), \text{ where } 0 < x < 200,$$

can be written as

$$\ln x = \ln 200 - Ae^{-at},$$

where A is an arbitrary positive constant. [5]

(v) Use the size of the wolf population at $t = 0$ to find the value of A. Give your answer in the form $A = \ln p$, where $p \in \mathbb{Z}^+$. [2]

(vi) Use the size of the wolf population at $t = 5$, given in part (a), to show that $a = 0.0855$, correct to three significant figures. [2]

(vii) Use the Gompertz model to predict the size of the wolf population at $t = 10$. Give your answer correct to the nearest whole number. [3]

After 10 years, the wolf population is measured and is found to be 85.

(c) Comment on the predictions made by the two models. [1]

By tracking individual wolves, the researchers find that about 3% of the wolf population emigrate from the protected area each year.

They decide to adapt the Gompertz model to allow for this. The new model will satisfy the differential equation

$$\frac{dx}{dt} = 0.0855x\ln\left(\frac{200}{x}\right) - 0.03x.$$

(d) (i) Use Euler's method, with a step size of 0.5 years and an initial value of $x_0 = 70$ when $t = 5$, to find an estimate for the size of the wolf population when $t = 10$. Give your answer correct to the nearest whole number. [4]

(ii) Comment on your answer. [1]