Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 3

14 November 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 15 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 26]

This question asks you to investigate lines normal to curves of the form $y=\frac{k^2}{x}$**.**

The curve H has equation $y=\frac{1}{x}$ where $x\in\mathbb{R}, x\neq 0$.

(a) A line N is normal to H at $x=t$.

(i) Show that the gradient of N is t^2. [2]

(ii) Hence, show that the equation of N is $y=t^2x+\frac{1}{t}-t^3$. [1]

(b) The equation for N given in part (a)(ii) is of the form $y=mx+c$.

(i) Show that either $c=\frac{1}{\sqrt{m}}\left(1-m^2\right)$ or $c=\frac{1}{\sqrt{m}}\left(m^2-1\right)$. [4]

(ii) Determine the set of values of m for which there exists at least one line normal to H. [1]

(c) Hence, or otherwise, determine the set of values of m for which there exists exactly

(i) one line normal to H; [1]

(ii) two lines normal to H. [2]

(d) On the same set of axes, sketch

(i) the curve H; [1]

(ii) for an appropriate value of m, two lines that satisfy the result found in part (c)(ii). Clearly indicate the point at which each line is normal to H. [2]

You are **not** required to state the equations of these lines nor determine where they intersect H or the coordinate axes.

(This question continues on the following page)

(Question 1 continued)

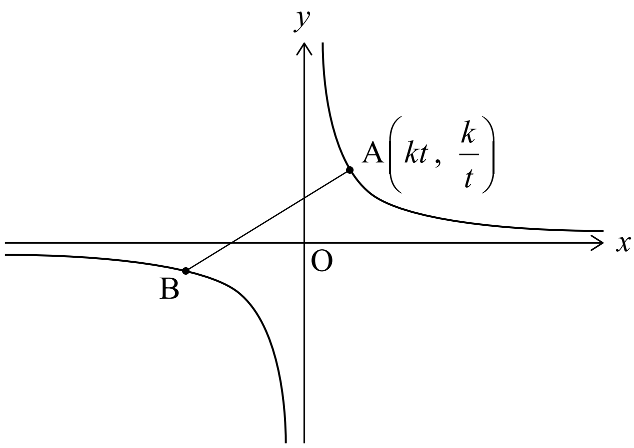
The curve F has equation $y=\frac{k^2}{x}$ where $x \in \mathbb{R}, x \neq 0$ and $k \in \mathbb{R}, k \neq 0$.

The point $\mathrm{A}\left(kt,\ \frac{k}{t}\right)$, where $t \in \mathbb{R}, t \neq \pm 1$, lies on F.

The line normal to F at A intersects F again at point B.

The line segment [AB] is shown in the following diagram.

diagram not to scale



(e) The equation of the line normal to F at A is given by $y=t^2x-kt^3+\frac{k}{t}$.

(i) Show that the x-coordinates of A and B satisfy the quadratic equation

$$x^2-k\left(t-\frac{1}{t^3}\right)x-\frac{k^2}{t^2}=0.$$ [3]

(ii) Hence, by considering either the sum or product of the roots of this quadratic equation, or otherwise, determine the coordinates of B. [3]

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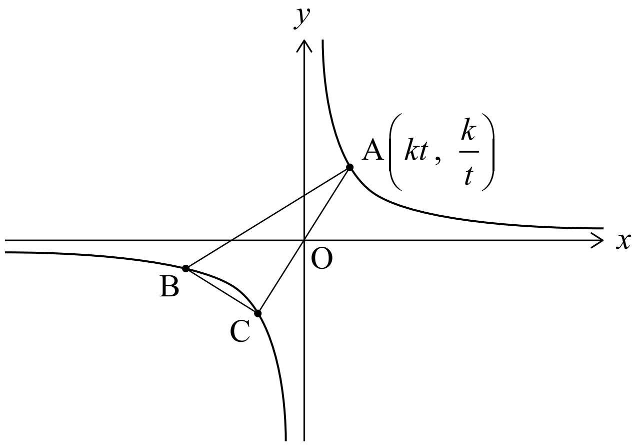
(Question 1 continued)

From A, the line passing through the origin O intersects F again at point C.

Points A, B and C form triangle ABC as shown in the following diagram.

diagram not to scale



(f) Prove that $\mathrm{B\hat{C}A}$ is a right angle. [6]

2. [Maximum mark: 29]

In this question researchers are trying to find the most accurate model to use when modelling a population of wolves.

Historically, a population of wolves in an area had a stable size of 200. After some years of disruption, the population was reduced to 40 wolves. At this point, the area became a protected space and the population began to grow again.

Researchers in the area wish to model the size of the wolf population, x, as a function of t, where t is the time, in years, since the area became protected.

(a) Initially, the researchers consider using the logistic model

$$x = \frac{L}{1 + Ce^{-kt}}, \text{ where } L, C, k \in \mathbb{R}^+.$$

The researchers decide to let $L = 200$.

(i) State the assumption being made in assuming $L = 200$. [1]

At $t = 0$, the population of wolves is 40.

(ii) Find the value of C. [2]

At $t = 5$, the population of wolves is found to have increased to 70.

(iii) Find the value of k. [2]

(iv) Use your model to predict the size of the wolf population in the area 10 years after it became protected. Give your answer correct to the nearest whole number. [2]

(b) An alternative model for population growth is called the Gompertz model. When applied by the researchers to the wolf population, this model satisfies the differential equation

$$\frac{dx}{dt} = ax\ln\left(\frac{200}{x}\right), \ a \in \mathbb{R}^+.$$

(i) Write down the value of $\frac{dx}{dt}$ when $x = 200$. [1]

(ii) Interpret your answer to part (b)(i) in context. [1]

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(Question 2 continued)

Consider the function $f(x) = \ln(\ln 200 - \ln x)$, where $0 < x < 200$.

(iii) Show that $f'(x) = \dfrac{-1}{x\ln\left(\dfrac{200}{x}\right)}$. [2]

(iv) Hence, use separation of variables to show that the general solution of

$$\frac{dx}{dt} = ax\ln\left(\frac{200}{x}\right), \text{ where } 0 < x < 200,$$

can be written as

$$\ln x = \ln 200 - Ae^{-at},$$

where A is an arbitrary positive constant. [5]

(v) Use the size of the wolf population at $t = 0$ to find the value of A.
Give your answer in the form $A = \ln p$, where $p \in \mathbb{Z}^+$. [2]

(vi) Use the size of the wolf population at $t = 5$, given in part (a), to show that $a = 0.0855$, correct to three significant figures. [2]

(vii) Use the Gompertz model to predict the size of the wolf population at $t = 10$.
Give your answer correct to the nearest whole number. [3]

After 10 years, the wolf population is measured and is found to be 85.

(c) Comment on the predictions made by the two models. [1]

By tracking individual wolves, the researchers find that about 3% of the wolf population emigrate from the protected area each year.

They decide to adapt the Gompertz model to allow for this. The new model will satisfy the differential equation

$$\frac{dx}{dt} = 0.0855x\ln\left(\frac{200}{x}\right) - 0.03x.$$

(d) (i) Use Euler's method, with a step size of 0.5 years and an initial value of $x_0 = 70$ when $t = 5$, to find an estimate for the size of the wolf population when $t = 10$.
Give your answer correct to the nearest whole number. [4]

(ii) Comment on your answer. [1]