Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

11 November 2025

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

(a) Show that $3\cos 2x + 11\sin x = 3 + 11\sin x - 6\sin^2 x$. [2]

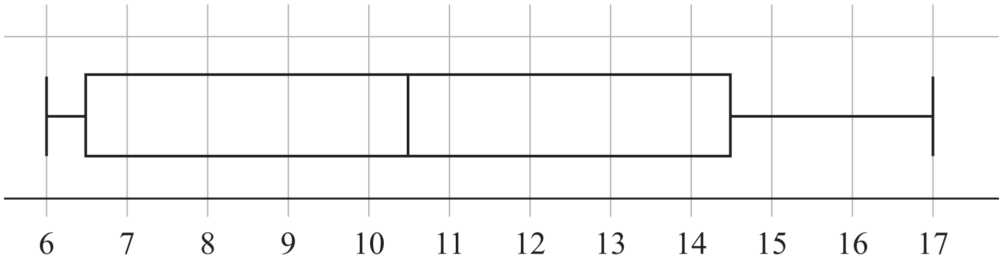
(b) Hence, or otherwise, solve the equation $3\cos 2x + 11\sin x - 6 = 0$ for $0° \le x \le 180°$. [3]

2. [Maximum mark: 6]

A teacher sets her class of 30 pupils a quiz.

Aiden and Brett were absent on the day of the quiz.

The following box and whisker diagram shows the results of the 28 pupils who took the quiz on the day.



quiz score

Aiden and Brett take the quiz when they return.

Aiden scores less than 6.

Brett scores more than 17.

(a) Explain, briefly, why the median score for all 30 pupils would still be 10.5. [1]

The mean score of the 28 pupils was 10.5.

The mean score for all 30 pupils is now 10.6.

The range of scores for all 30 pupils is 14.

(b) Determine Aiden's score and Brett's score. [5]

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3. [Maximum mark: 6]

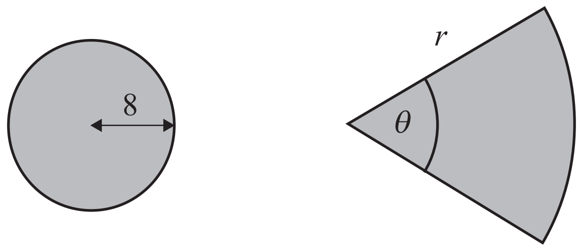
Consider a circle and a sector.

The radius of the circle is $8\,\text{mm}$.

The radius of the sector is $r\,\text{mm}$ and the acute angle at the centre is θ radians.

This is shown in the following diagram.

diagram not to scale



The perimeter of the sector is 1.5 times the circumference of the circle.

(a) Show that $r = \dfrac{24\pi}{2+\theta}$. [3]

It is given that the area of the circle is the same as the area of the sector.

(b) Determine the value of θ. [3]

4. [Maximum mark: 7]

A garden centre sells seeds for two varieties of sunflowers: large and giant.

The heights of large sunflowers are normally distributed with mean μ and standard deviation σ.

It is known that 25% of large sunflowers have a height of over 180 cm.

(a) Given that $\mu + A\sigma = 180$, where $A \in \mathbb{R}$, find the value of A. [3]

The heights of giant sunflowers are also normally distributed.

The giant sunflowers have a mean height that is 35 cm more than that of the large sunflowers and a standard deviation that is double that of the large sunflowers.

It is known that 98% of giant sunflowers have a height greater than 180 cm.

(b) Determine μ and σ. [4]

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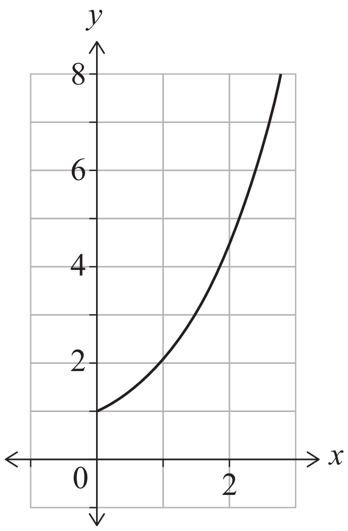
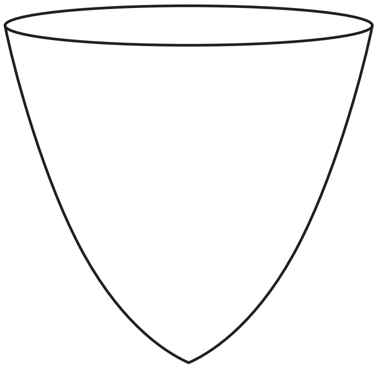
5. [Maximum mark: 5]

Consider the function $f(x) = e^{\frac{3x}{4}}$, where $x \geq 0$.

The region enclosed by the curve $y = f(x)$, the line $y = 8$ and the y-axis is rotated by 2π radians about the y-axis to form a solid. This is shown in the following diagrams.





Find the volume of the solid formed.

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6. [Maximum mark: 7]

Consider the quadratic polynomial given by $P(x) = 2x^2 + qx + r$ where $q, r \in \mathbb{R}$.

The equation $P(x) = 0$ has roots α and β, where $\alpha, \beta \in \mathbb{R}$.

When $P(x)$ is divided by $(x + 1)$, the remainder is 12.

Given that $\alpha^2 + \beta^2 = \frac{37}{4}$, find the value of q and the value of r.

7. [Maximum mark: 6]

Consider the following two equations.

$$2x + 6y - 8z = 13$$

$$3x - y + 3z = 12$$

(a) Determine the general solution giving the answer in parametric form. [2]

Consider a third equation $ax + 12y - 16z = k$, where $a, k \in \mathbb{R}$.

(b) In the case where $a = 3$ and $k = -3$, determine the unique solution to the system of three equations. [2]

(c) (i) Write down the value of a for which there is no unique solution to the system of three equations.

(ii) Hence, write down the corresponding value of k for which there is an infinite number of solutions to the system of three equations. [2]

8. [Maximum mark: 7]

Malik owns 15 recipe books which he bought while travelling.

(a) Find the number of different ways of arranging the books in a line on a shelf. [1]

Malik's books were bought in three continents.

Six books are from Asia, five from Europe and four from Africa.

Malik decides to arrange the books on the shelf so that the books from each of the continents are grouped together.

(b) Determine the number of different ways that Malik can do this. [3]

Malik chooses four books, all from the same continent.

(c) Determine the number of different choices Malik could make. [3]

9. [Maximum mark: 5]

Prove by contradiction that $\sqrt{2xy} \le \frac{1}{2}(2x+y)$ for all $x, y \in \mathbb{R}^{+}$.

Do **not** write solutions on this page.

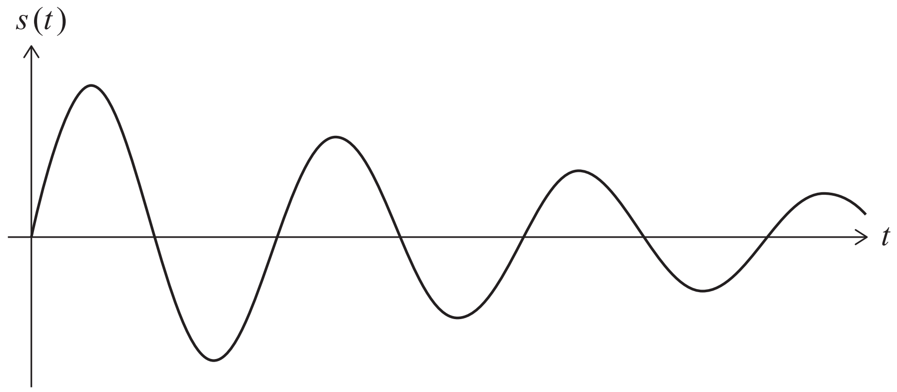
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

A particle P moves in a straight line so that its displacement, $s\,\text{cm}$, from a fixed point O at time t seconds is given by $s(t) = 2^{\left(1-\frac{t}{5}\right)}\sin\left(\frac{2\pi t}{3}\right)$, where $t \geq 0$.

The following diagram shows part of the graph of $y = s(t)$.



(a) Find

(i) the maximum displacement of P from O;

(ii) the maximum velocity of P. [5]

(b) Find

(i) the minimum value of the displacement function $s(t)$;

(ii) the displacement of P from O when $t = 3.5$. [3]

(c) Hence, determine the **total distance** travelled by P in the first 3.5 seconds. [3]

The first time that P returns and passes through O is when $t = T$.

(d) Write down the value of T. [1]

(This question continues on the following page)

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(Question 10 continued)

The particle passes through O every T seconds.

A sequence $u_1, u_2, u_3 \dots$ is formed where $u_1, u_2, u_3 \dots$ are the largest **distances** from O in each of the intervals $0 < t < T,\ T < t < 2T,\ 2T < t < 3T \dots$ respectively.

It is known that $u_1, u_2, u_3 \dots$ form a geometric sequence.

(e) (i) Determine the value of the common ratio r of this geometric sequence.

(ii) Calculate the **total distance** travelled by the particle if it were to continue to move in this way indefinitely. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

The lengths, in metres, of jumps in a long jump competition can be modelled by a continuous random variable X with probability density function f defined by:

$$f(x)=\begin{cases} 0, & x<3 \\ a(x-3)^3+b(x-3)^2, & 3\le x\le 9 \\ 0, & x>9 \end{cases}$$

where $a, b \in \mathbb{R}$, $a, b \neq 0$.

(a) Show that $324a + 72b = 1$. [4]

It is given that $f(9) = 0$.

(b) (i) Show that $6a + b = 0$.

(ii) Determine the value of a and the value of b. [4]

(c) Show that the median of X is less than the mode of X. [5]

Matt has the final jump in the competition. He needs to jump at least $8.52\,\text{m}$ to win the competition.

(d) Given that he jumps over $8\,\text{m}$, use the model to find the probability that he wins the competition. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 22]

Consider the following homogenous differential equation

$$\left(x^2+xy\right)\frac{\mathrm{d}y}{\mathrm{d}x}=x^2+xy-3y^2 \text{ where } x>0 \text{ and } y>\frac{x}{2}.$$

It is given that $y=\frac{3}{2}$ when $x=1$.

(a) (i) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x=1$.

(ii) Use Euler's method with two equal steps to estimate the value of y when $x=1.4$.

(iii) Hence, state the concavity of the solution curve for $1\le x\le 1.4$. You may assume that the concavity does not change in this interval. Give a reason for your answer. [6]

(b) (i) Show that

$$\left(x^2+xy\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2x+y-x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-(x+7y)\frac{\mathrm{d}y}{\mathrm{d}x}.$$

(ii) Find the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $x=1$. [6]

(c) Determine constants A, $B\in\mathbb{R}$ such that $\frac{1+v}{1-4v^2}\equiv\frac{A}{1-2v}+\frac{B}{1+2v}$. [2]

(d) By solving the differential equation $\left(x^2+xy\right)\frac{\mathrm{d}y}{\mathrm{d}x}=x^2+xy-3y^2$ where $x>0$, $y>\frac{x}{2}$ and $y=\frac{3}{2}$ when $x=1$, show that $x^6(2y-x)^3=2(x+2y)$. [8]

Please **do not** write on this page.

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