Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

10 November 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

14 pages
8825 – 7116

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The 7th term of an arithmetic sequence is 6.

The sum of the 6th term and the 12th term is 24.

Find the first term and the common difference.

2. [Maximum mark: 6]

Events A and B are such that $\mathrm{P}(A)=\frac{1}{2}$, $\mathrm{P}(B)=\frac{1}{3}$ and $\mathrm{P}(A\,|\,B)=\frac{1}{4}$.

(a) Find the probability that both events A and B occur. [2]

(b) Find $\mathrm{P}(A\,|\,B')$. [4]

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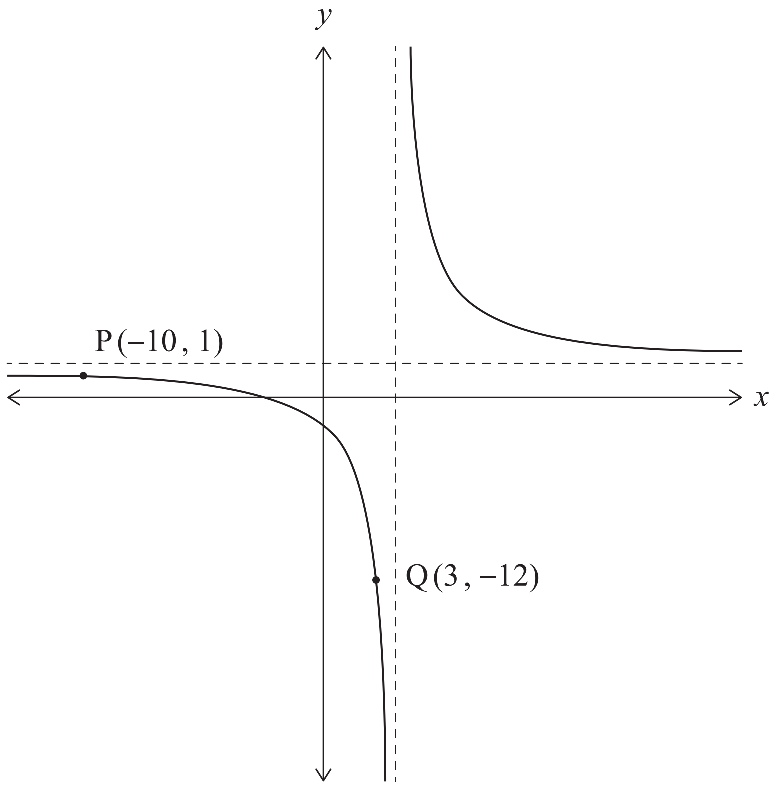
3. [Maximum mark: 8]

The following diagram shows the graph of $y = \dfrac{Ax + B}{x - 4}$, where $x \in \mathbb{R}, x \neq 4$ and $A, B \in \mathbb{Z}$.

The graph passes through the points $P(-10, 1)$ and $Q(3, -12)$.



(a) Determine the value of A and the value of B. [3]

(b) Describe a sequence of transformations that would map the graph of $y = \dfrac{1}{x}$ onto the graph of $y = \dfrac{Ax + B}{x - 4}$. [5]

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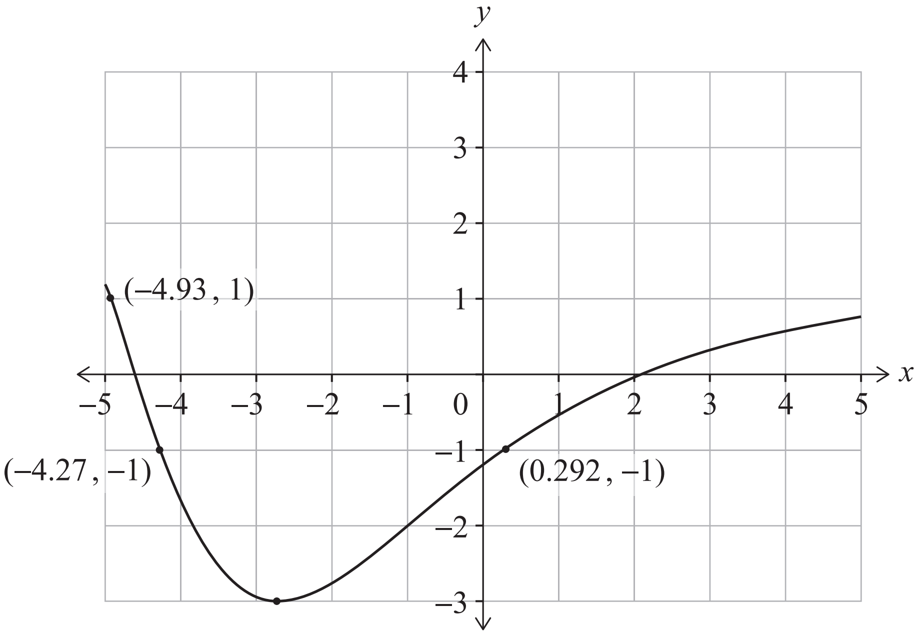
4. [Maximum mark: 5]

Solve the equation $3\log_8 10x - \log_4 x = 1$ for $x > 0$.

5. [Maximum mark: 8]

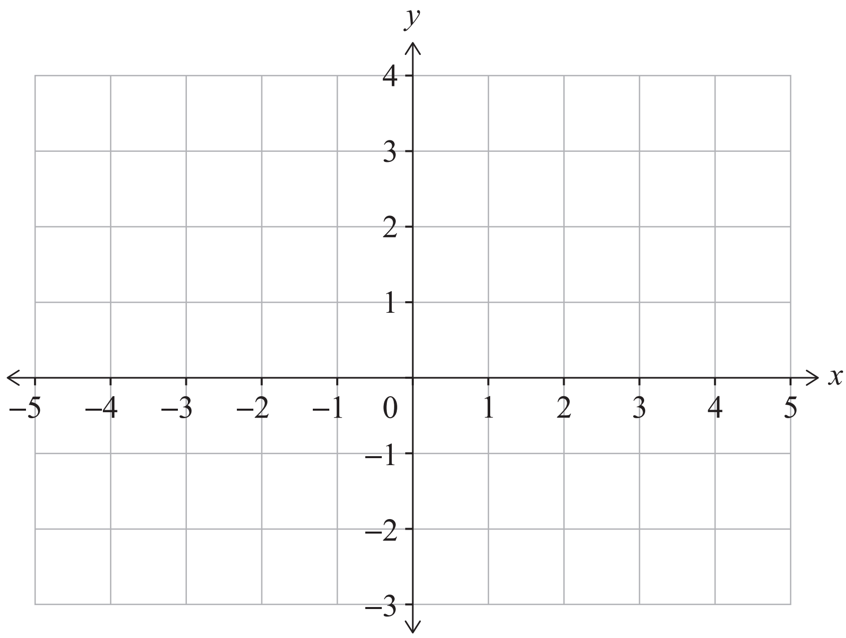
The graph of $y = f(x)$ for $-5 \le x \le 5$ is shown in the following diagram.

The curve passes through the points $(-4.93, 1)$, $(-4.27, -1)$ and $(0.292, -1)$.



(a) On the following set of axes,

(i) sketch the graph $y = f(|x|)$ showing the approximate intercepts with the axes;



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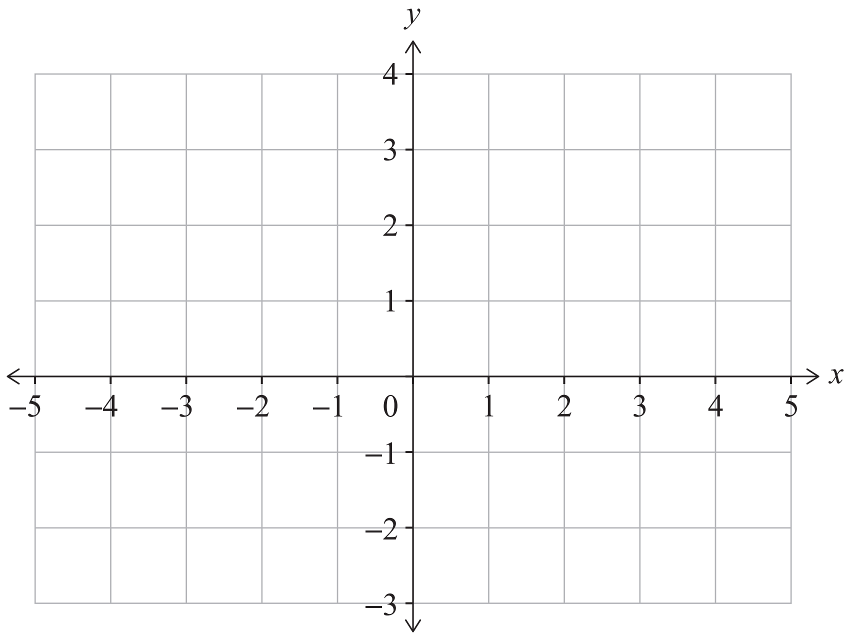
(Question 5 continued)

(ii) sketch the graph $y = |f(x)|$ showing the approximate intercepts with the axes. [4]



(b) Solve the inequality $|f(x)| \geq (f(x))^2$. [4]

6. [Maximum mark: 7]

In this question you may assume the result $\int x\operatorname{cosec}^2 x\,\mathrm{d}x = \ln|\sin x| - x\cot x + c$.

Consider the differential equation $x\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = \frac{\operatorname{cosec}^2 x}{x}$ for $0 < x < \pi$.

Given that $y = \frac{8}{\pi^2}$ when $x = \frac{\pi}{2}$, find the particular solution for this differential equation.

Express your answer in the form $y = f(x)$.

7. [Maximum mark: 8]

The discrete random variable X has the following probability distribution:

x	1	2	3
$\mathrm{P}(X=x)$	$0.6-2a$	$3a$	$0.4-a$

(a) Find the range of possible values of a. [2]

(b) In the case where $a=0.2$, determine $\mathrm{Var}(2-X)$. [6]

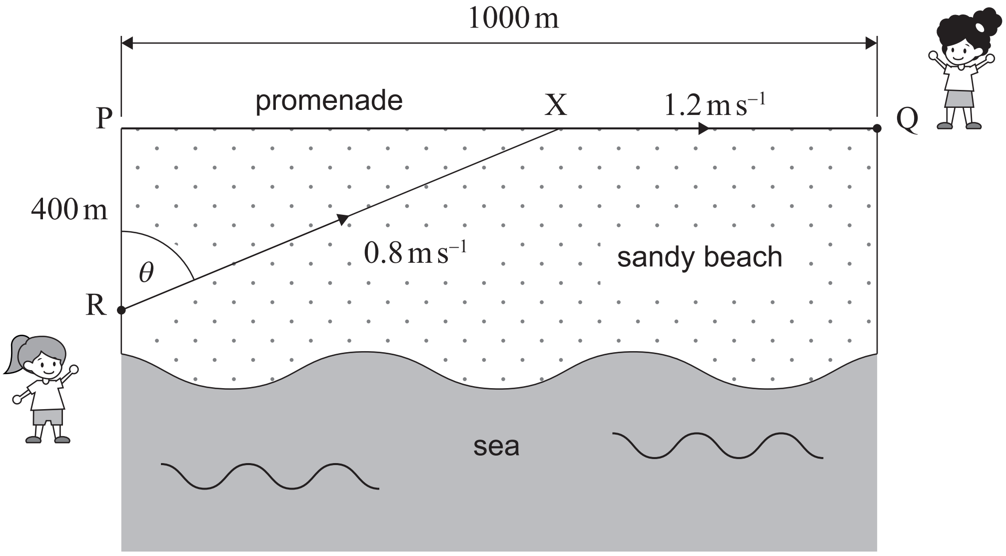
8. [Maximum mark: 9]

Astrid and Bronwyn are on vacation at Blackpool beach.

Astrid is standing on the beach at point R and sees Bronwyn standing on the promenade at point Q. $\mathrm{PR} = 400\,\mathrm{m}$, $\mathrm{PQ} = 1000\,\mathrm{m}$, $\mathrm{P\hat{R}X} = \theta$ where $0 < \tan\theta \le \frac{5}{2}$.

Astrid walks in a straight line from R with speed $0.8\,\mathrm{m\,s^{-1}}$ until reaching point X on the promenade. She then jogs along the promenade with speed $1.2\,\mathrm{m\,s^{-1}}$.

This is shown in the following diagram.



It is given that $\mathrm{RX} = 400\sec\theta$ and $\mathrm{XQ} = 1000 - 400\tan\theta$.

The time, in seconds, for Astrid to reach Bronwyn is given by T.

(a) Show that $T = 500\sec\theta + \dfrac{2500 - 1000\tan\theta}{3}$. [2]

(b) Find $\dfrac{\mathrm{d}T}{\mathrm{d}\theta}$. [3]

Astrid chooses the angle θ of her walk across the beach to point X, in order to reach Bronwyn in the shortest time possible. You may assume that T has exactly one minimum value.

(c) Show that in this case $\mathrm{PX} = 160\sqrt{5}\,\mathrm{m}$. [4]

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(Question 8 continued)

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

Consider the function defined by $f(x) = \frac{1}{2}x^2 + kx + 13$, where $x \in \mathbb{R}$ and $k \in \mathbb{Z}^+$.

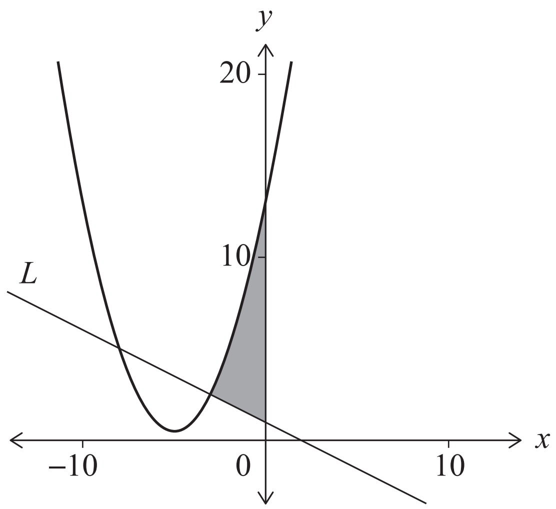
(a) Given that the equation $f(x) = 0$ has no real roots, show that the greatest possible value of k is 5. [2]

For the remainder of this question, consider the case $k = 5$.

(b) (i) Write down the equation of the axis of symmetry of the graph of f.

(ii) Hence, or otherwise, determine the coordinates of the minimum point on the graph of f. [3]

The following diagram shows the graph of f and a line L which is normal to the curve at $x = -3$. The shaded area shown is bounded by the curve, the line L and the y-axis.



(c) Show that the equation of L is given by $y = -\frac{1}{2}x + 1$. [5]

(d) Hence, find the shaded area. [5]

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10. [Maximum mark: 19]

The point $\mathrm{P}(-1, 1, -13)$ lies on the line L_1. The line L_1 has a direction vector $\begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix}$.

(a) Write down a vector equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$. [1]

(b) Find a vector equation for line L_2 in the form $\boldsymbol{s} = \boldsymbol{c} + \mu\boldsymbol{d}$, given that L_2 passes through the points $\mathrm{A}(2, -4, 2)$ and $\mathrm{B}(7, -6, 1)$. [2]

(c) Show that L_1 and L_2 are skew. [5]

The point N lies on L_2.

(d) Find $\overrightarrow{\mathrm{PN}} \cdot \overrightarrow{\mathrm{AB}}$ in terms of μ. [4]

(e) Given that N is the point on L_2 that lies closest to P, find the coordinates of N. [3]

(f) Point O denotes the origin $(0, 0, 0)$.
Find the equation of the plane containing points O, P and N, giving your answer in the form $\alpha x + \beta y + \gamma z = \delta$, where $\alpha, \beta, \gamma, \delta \in \mathbb{Z}$. [4]

Do **not** write solutions on this page.

11. [Maximum mark: 20]

(a) It is given that $w = 1 + \mathrm{i}\sqrt{3}$.

(i) Express w in the form $w = r\mathrm{e}^{\mathrm{i}\alpha}$, where $r > 0$ and $-\pi < \alpha \leq \pi$.

(ii) Show that $\left(1+\mathrm{i}\sqrt{3}\right)^8 + \left(1-\mathrm{i}\sqrt{3}\right)^8 = -256$. [6]

It is given that $\left(1+\mathrm{i}\tan\theta\right)^n \equiv \dfrac{\cos n\theta + \mathrm{i}\sin n\theta}{\cos^n\theta}$ for $\cos\theta \neq 0$, $n \in \mathbb{Z}^+$.

(b) Show that $\left(1+\mathrm{i}\tan\theta\right)^4 + \left(1-\mathrm{i}\tan\theta\right)^4 \equiv \dfrac{2\cos 4\theta}{\cos^4\theta}$. [3]

(c) Hence, determine the four roots of the equation $(1+z)^4 + (1-z)^4 = 0$, giving your answers in the form $z = \mathrm{i}\tan\left(\dfrac{k\pi}{8}\right)$, where $k \in \mathbb{Z}^+$. [4]

(d) By considering the binomial expansion of $(1+z)^4 + (1-z)^4$, show that $\tan\dfrac{\pi}{8} = \sqrt{3-2\sqrt{2}}$. [7]

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