8825 – 7106M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2025

Mathematics: analysis and approaches

Higher level

Paper 1

28 pages

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

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Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

Section A

1. (a) attempt to find the common difference ***(M1)***

$12 = 36 + 4d$ OR $\frac{12-36}{5-1}$

$d = -6$ ***(A1)***

attempt to use the term formula for an arithmetic sequence ***(M1)***

$u_{13} = u_1 + 12(-6)$ OR $u_5 + 8(-6)$

$u_{13} = -36$ ***A1***

[4 marks]

(b) **METHOD 1**

attempt to set $S_n = 0$ ***(M1)***

$\frac{n}{2}\left(2(36)+(n-1)(-6)\right)=0$ OR $\frac{n(36+u_n)}{2}=0 \Rightarrow u_n = -36$

$n = 13$ ***A1***

Note: Condone $n = 0, 13$

METHOD 2

attempt to recognize symmetry ***(M1)***

$36 + 30 + ... + 0 + ... - 30 - 36$

$n = 13$ ***A1***

[2 marks]

Total [6 marks]

2. (a) attempt to factorize at least one expression ***(M1)***

$$\frac{x}{(x-7)(x-1)}\times\frac{(x-1)(x+1)}{x+1} \quad \textbf{OR} \quad \frac{x(x^2-1)}{(x-7)(x^2-1)} \quad \textbf{OR} \quad \frac{x(x^2-1)}{(x-7)(x-1)(x+1)}$$ ***A1***

$$\frac{x}{x-7}$$ ***AG***

[2 marks]

(b) **METHOD 1** (combines given log terms)

attempt to use the subtraction/quotient property of logs ***(M1)***

$$\log_2\left(\frac{x\left(x^2-1\right)}{\left(x^2-8x+7\right)(x+1)}\right)$$

valid attempt to rewrite equation with (or without) logs on both sides (seen anywhere) ***(M1)***

$$2=\frac{x}{x-7} \quad \textbf{OR} \quad \log_2\left(\frac{x}{x-7}\right)=\log_2 2 \quad \textbf{OR} \quad 2^{\log_2\left(\frac{x}{x-7}\right)}=2^1$$ ***(A1)***

$x=14$ ***A1***

METHOD 2 (uses log laws with LHS)

attempt to use $1=\log_2 2$ (seen anywhere) ***(M1)***

attempt to use the subtraction/quotient property of logs ***(M1)***

$$\log_2\left(\frac{x^3-x}{2}\right)=\log_2\left(\left(x^2-8x+7\right)(x+1)\right) \quad \textbf{OR} \quad \frac{x\left(x^2-1\right)}{2}=\left(x-1\right)\left(x-7\right)(x+1)$$ ***(A1)***

$$\frac{x}{2}=x-7$$

$x=14$ ***A1***

[4 marks]

Total [6 marks]

3. (a) (i) $c = 9$ ***A1***

(ii) attempt to substitute (3,0) into $y = ax^2 + c$ with *their* c ***(M1)***

$9a + 9 = 0$

$a = -1$ ***A1***

[3 marks]

(b) **METHOD 1** (single integral)

attempt to subtract areas (seen anywhere) ***(M1)***

$\int_0^3 (9 - x^2) - (9 - 3x)\ \mathrm{d}x \quad \left(= \int_0^3 3x - x^2\ \mathrm{d}x\right)$ OR $\int_0^3 (9 - x^2)\mathrm{d}x - \text{triangle area}$

correct integration $\left[\frac{3}{2}x^2 - \frac{1}{3}x^3\right]_0^3$ **OR** $\left[9x - \frac{1}{3}x^3\right]_0^3$ ***(A1)***

attempt to substitute their limits into their integrated expression (condone absence of the 0 substitution) ***(M1)***

$\frac{3}{2}(3)^2 - \frac{1}{3}(3)^3 - \left(\frac{3}{2}(0)^2 - \frac{1}{3}(0)^3\right)$ OR $9(3) - \frac{1}{3}(3)^3$

$\frac{27}{2} - 9$ OR $18 - \frac{27}{2}$

Area $= \frac{9}{2}$ (square units) ***A1***

continued…

Question 3 continued.

METHOD 2 (evaluating two integrals separately)

$\int_0^3 (9-x^2)\,\mathrm{d}x = \left[9x - \frac{1}{3}x^3\right]_0^3$ **OR** $\int_0^3 (9-3x)\,\mathrm{d}x = \left[9x - \frac{3}{2}x^2\right]_0^3$ ***(A1)***

Note: Award ***A1*** for correct integration. Ignore absence of limits.

attempt to substitute their limits into both of their integrated expressions (condone absence of the 0 substitution) ***(M1)***

$9(3) - \frac{1}{3}(3)^3 - \left(9(0) - \frac{1}{3}(0)^3\right)$ and $9(3) - \frac{3}{2}(3)^2$

attempt to subtract the two integrals in any order (seen anywhere) ***(M1)***

$9(3) - \frac{1}{3}(3)^3 - \left(9(3) - \frac{3}{2}(3)^2\right)$ OR $\int (9-x^2)\,\mathrm{d}x - \int (9-3x)\,\mathrm{d}x$

$18 - \frac{27}{2}$

Area $= \frac{9}{2}$ (square units) ***A1***

[4 marks]

Total [7 marks]

4. (a) **METHOD 1**

x-coordinate of the vertex at A is 2 ***(A1)***

attempt to substitute their x-coordinate into equation of line ***(M1)***

$2(2)-1(=3)$

$3=-(2)^2+4(2)+p$ ***A1***

$p=-1$ ***AG***

METHOD 2

x-coordinate of the vertex at A is 2 ***(A1)***

attempt to substitute their x-coordinate into $f(x)$ ***(M1)***

$f(2)=p+4$

$p+4=2(2)-1$ ***A1***

$p=-1$ ***AG***

METHOD 3

attempt to complete the square on $f(x)$ ***(M1)***

$-(x-2)^2+p+4$

$x_{\text{A}}=2$ **OR** $y_{\text{A}}=p+4$ ***(A1)***

$p+4=2(2)-1$ ***A1***

$p=-1$ ***AG***

Note: Award no marks if $x=0$ is substituted into $-x^2+4x+p=2x-1$ to show $p=-1$.

[3 marks]

continued…

Question 4 continued.

(b) **METHOD 1** (equating $g(x)$ and $2x-1$)

x-coordinate is $\frac{-q}{2}$ (seen anywhere) ***(A1)***

attempt to find intersection points, equating $g(x)$ and $2x-1$ ***(M1)***

$x^2+qx-1=2x-1$

attempt to substitute their x-coordinate ***(M1)***

$\left(\frac{-q}{2}\right)^2+q\left(\frac{-q}{2}\right)-1=2\left(-\frac{q}{2}\right)-1$

$q^2-4q=0$ ***(A1)***

$q=4$ (since $q\neq 0$) ***A1***

METHOD 2 (find coordinates of B in terms of q)

x-coordinate is $\frac{-q}{2}$ ***(A1)***

EITHER

attempt to substitute their x-coordinate into $y=2x-1$ ***(M1)***

$2\left(\frac{-q}{2}\right)-1 \Rightarrow \mathrm{B}\left(\frac{-q}{2},-q-1\right)$

attempt to substitute their coordinates of B into $g(x)$ ***(M1)***

$-q-1=\left(\frac{-q}{2}\right)^2+q\left(\frac{-q}{2}\right)-1$

OR

attempt to substitute their x-coordinate into $g(x)$ ***(M1)***

$\left(\frac{-q}{2}\right)^2+q\left(\frac{-q}{2}\right)-1 \Rightarrow \mathrm{B}\left(\frac{-q}{2},-\frac{q^2}{4}-1\right)$

attempt to substitute their coordinates of B into the line ***(M1)***

$-\frac{q^2}{4}-1=2\left(\frac{-q}{2}\right)-1$

continued…

Question 4 continued.

THEN

$q^2 - 4q = 0$ ***(A1)***

$q = 4$ (since $q \neq 0$) ***A1***

METHOD 3 (completing the square)

attempt to complete the square on $g(x)$ ***(M1)***

$$\left(x + \frac{q}{2}\right)^2 - \left(\frac{q}{2}\right)^2 - 1$$

$x_{\mathrm{B}} = \frac{-q}{2}$ **OR** $y_{\mathrm{B}} = -\frac{q^2}{4} - 1$ ***(A1)***

attempt to substitute their coordinates of B into equation of line ***(M1)***

$$-\frac{q^2}{4} - 1 = 2\left(\frac{-q}{2}\right) - 1$$

$q^2 - 4q = 0$ ***(A1)***

$q = 4$ (since $q \neq 0$) ***A1***

[5 marks]

Total [8 marks]

5. (a) (i) **METHOD 1**

$f'(x) = 4(1-x)^{-5}$ ***A1***

substitution of x=0 into derivative ***M1***

$4(1-0)^{-5}$ OR $f'(0) = 4$

$a = 4$ ***AG***

Note: Do not award ***M1*** for $1+4x$ seen without clear evidence of a substitution.

METHOD 2

attempt to create binomial expansion to at least two terms ***M1***

$$1+\binom{-4}{1}x+\left(\binom{-4}{2}(x)^2+\ldots\right)$$

correct expansion

$$1+(-4)(-x)+\left(\frac{(-4)(-5)}{2}(-x)^2+\ldots\right)$$ ***A1***

$a = 4$ ***AG***

Note: Do not award ***A1*** for $1+4x$ seen without clear evidence of correct working.

(ii) **METHOD 1**

$f''(x) = (-5)\times(4)(1-x)^6\times(-1),\quad 20(1-x)^{-6}$ ***(A1)***

$(f''(0) =)20$ ***(A1)***

$\frac{20}{2!}$

$b = 10$ ***A1***

METHOD 2

$\frac{(-4)(-5)}{2!}(-x)^2$ (may be seen in their expansion in (i)) ***(A1)***

$\frac{20}{2}$ ***(A1)***

$b = 10$ ***A1***

[5 marks]

continued…

Question 5 continued.

(b) attempt to substitute 0.1 into the expansion ***(M1)***

1.52

1520 (dollars) ***A1***

[2 marks]

Total [7 marks]

6. **METHOD 1 Row Reduction**

attempt to reduce a matrix or eliminate one variable ***(M1)***

two correct rows or equivalent equations ***A1A1***

$$\begin{bmatrix} 1 & 0 & -1 & 4 \\ 2 & 2 & 1 & 14 \\ 1 & 2 & \alpha & \beta \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 4 \\ 0 & 2 & 3 & 6 \\ 0 & 2 & 1+\alpha & \beta - 4 \end{bmatrix}$$

correct third row or equivalent equation ***A1***

$$\begin{bmatrix} 1 & 0 & -1 & 4 \\ 0 & 2 & 3 & 6 \\ 0 & 0 & \alpha-2 & \beta-10 \end{bmatrix}$$

equating the third row coefficients to zero ***(M1)***

$\alpha = 2$ and $\beta = 10$ ***A1***

METHOD 2 Writing an equation in one variable

attempt to eliminate either x or z using the first two equations (seen anywhere) ***(M1)***

$2(z+4)+2y+z=14$ OR $2x+2y+(x-4)=14$

one correct equation ***A1***

$2y+3z=6$ **OR** $3x+2y=18$

correct substitution into the third equation to create an equation in either x or z ***A2***

$x+18-3x+\alpha(x-4)=\beta$ **OR** $z+4+6-3z+\alpha z=\beta$

OR $(\alpha-2)x=\beta+4\alpha-18$ **OR** $z(\alpha-2)=\beta-10$

attempt to set coefficients equal to zero for infinite solutions ***(M1)***

$\alpha = 2$ and $\beta = 10$ ***A1***

continued…

Question 6 continued.

METHOD 3 Subtracting and comparing equations

attempt to add or subtract two equations ***(M1)***

$x+2y+2z=10$ **OR** $2x+2y+(\alpha-1)z=\beta+4$ **OR** $x+(1-\alpha)z=14-\beta$ ***A2***

attempt to compare coefficients with the third equation ***(M1)***

$\alpha-1=1$, $\beta+4=14$, $1-\alpha=-1$, $14-\beta=4$

$\alpha=2$ and $\beta=10$ ***A1A1***

METHOD 4 Vector approach

attempt to find direction vector of line of intersection using π_1 and π_2 ***(M1)***

$$\begin{pmatrix}1\\0\\-1\end{pmatrix}\times\begin{pmatrix}2\\2\\1\end{pmatrix}$$

$$\left(\vec{d}=\right)\begin{pmatrix}2\\-3\\2\end{pmatrix}$$ ***A1***

normal of π_3 is perpendicular to $\begin{pmatrix}2\\-3\\2\end{pmatrix}$ ***(M1)***

$$\begin{pmatrix}1\\2\\\alpha\end{pmatrix}\bullet\begin{pmatrix}2\\-3\\2\end{pmatrix}=0$$

$\alpha=2$ ***A1***

correct substitution of a point common to the first two planes into the third plane ***A1***

$0+2(9)+2(-4)$, $4+2(3)+2(0)$

$\beta=10$ ***A1***

continued…

Question 6 continued.

METHOD 5 Finding a general point on the line of intersection

attempt to find intersection of π_1 and π_2 ***(M1)***

$x = 4 + z,\ \ y = 3 - \frac{3}{2}z$

$x = 4 + 2t,\ \ y = 3 - 3t,\ \ z = 2t$ **OR** $\mathbf{r} = (4,3,0) + t(2,-3,2)$ ***A1A1***

correct substitution of general point into π_3 ***A1***

$(4 + 2t) + 2(3 - 3t) + \alpha(2t),\quad 10 + t(2\alpha - 4)$

recognizing that the coefficient of t must be zero OR the constant term must be β ***(M1)***

$2\alpha - 4 = 0$ OR $\beta = 10$

$\alpha = 2$ and $\beta = 10$ ***A1***

Total [6 marks]

7. (a) attempt to separate variables ***(M1)***

$$\int \frac{\mathrm{d}y}{y^2} = \int \cos^2 x \ \mathrm{d}x$$

attempt to use an identity for $\cos^2 x$, may be seen above ***(M1)***

$$\frac{1}{2}(1+\cos 2x)$$

$$-y^{-1} = \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C$$ ***A1A1***

Note: Award ***A1*** for LHS and award ***A1*** for RHS. Award ***A1A0*** for at least one correct side and missing constant of integration.

$$-y^{-1} = \frac{2x+\sin 2x}{4} + C_1 \quad \textbf{OR} \quad y = \frac{-4}{2x+\sin 2x + C_2}$$ ***(A1)***

Note: Condone solutions that do not deal correctly with the constant(s) of integration.

$a = -4$ ***A1***

[6 marks]

(b) attempt to substitute initial conditions with *their* a. ***(M1)***

$$1 = \frac{-4}{\pi + 0 + b}$$

$b = -4 - \pi$ ***A1***

[2 marks]

Total [8 marks]

8. (a) **METHOD 1 Using the angle sum formula**

attempt to use angle sum formula ***M1***

$\sin 2\theta \cos \theta + \cos 2\theta \sin \theta$ ***A1***

attempt to use a double angle identity or the Pythagorean identity ***M1***

$2\sin\theta\cos^2\theta + \left(1-2\sin^2\theta\right)\sin\theta$ OR $2\sin\theta\left(1-\sin^2\theta\right)+\cos 2\theta\sin\theta$

$2\sin\theta\left(1-\sin^2\theta\right)+\left(1-2\sin^2\theta\right)\sin\theta$ (or equivalent) ***A1***

$3\sin\theta - 4\sin^3\theta$ ***AG***

METHOD 2 Using de Moivre's Theorem and comparing coefficients

$(\cos\theta + \mathrm{i}\sin\theta)^3 = \cos 3\theta + \mathrm{i}\sin 3\theta$ ***A1***

attempt to expand LHS ***M1***

$\cos^3\theta - 3\cos\theta\sin^2\theta + \mathrm{i}\left(3\cos^2\theta\sin\theta - \sin^3\theta\right)$

comparing imaginary parts ***M1***

$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$ ***A1***

$\left(\sin 3\theta =\right) 3\sin\theta - 4\sin^3\theta$ ***AG***

[4 marks]

(b) attempt to rewrite expression using result of part (a) ***(M1)***

$y = 2\sin 3\left(\theta + \frac{\pi}{6}\right),\ y = 2\sin\left(3\theta + \frac{\pi}{2}\right)$

vertical stretch factor 2 (seen anywhere) ***A1***

horizontal stretch factor $\frac{1}{3}$ followed by horizontal translation $\frac{\pi}{6}$ to the left ***A2***

OR

horizontal translation $\frac{\pi}{2}$ to the left followed by horizontal stretch factor $\frac{1}{3}$ ***A2***

Note: Award ***A1A0*** for correct horizontal transformations specified in the wrong order.

[4 marks]

Total [8 marks]

Section B

9. (a) recognition that $f'(x) = 0$ (seen anywhere) ***(M1)***

attempt to solve quadratic equation (condone not seeing $=0$) ***(M1)***

$a = -5,\ b = 1$ ***A1***

[3 marks]

(b) (local) maximum ***A1***

($f'(x)$ /gradient) changes from positive to negative (at $x = a$)

OR $f(x)$ changes from increasing to decreasing (at $x = a$)

OR $f''(a) = -18$ (accept $f''(a) < 0$) ***R1***

Note: Do not award ***A0R1***.

[2 marks]

(c) $f''(x) = 6x + 12$ ***(A1)***

$6c + 12 = 0$ ***(A1)***

$c = -2$ ***A1***

[3 marks]

(d) yes (there is a point of inflexion at $x = c$) ***A1***

($f''(x)$ /second derivative) changes sign at $x = c$

OR concavity (of f) changes at $x = c$

OR $f'(x)$ /gradient decreasing to the left of c, and increasing to the right of c ***R1***

[2 marks]

Note: Award ***R0*** if their only reason is that $f''(c) = 0$. Do not award ***A0R1***.

continued…

Question 9 continued.

(e) recognize to find $\int f'(x)\mathrm{d}x$ ***(M1)***

$x^3 + 6x^2 - 15x\left(+C\right)$ ***(A1)***

substitution of $\left(-2, 36\right)$ into their integrated expression (must contain $+C$) ***(M1)***

$36 = \left(-2\right)^3 + 6\left(-2\right)^2 - 15\left(-2\right) + C$

$C = -10$ ***A1***

$f(x) = x^3 + 6x^2 - 15x - 10$

[4 marks]

Total [14 marks]

10. (a) (i) attempt to create binomial expansion to four terms with $n=-1$ ***(M1)***

$$1+\binom{-1}{1}x^2+\binom{-1}{2}\left(x^2\right)^2+\binom{-1}{3}\left(x^2\right)^2+\ldots$$

$$1-x^2+\frac{(-1)(-2)}{2}\left(x^2\right)^2+\frac{(-1)(-2)(-3)}{6}\left(x^2\right)^3$$ ***A1A1***

Note: Award ***A1*** for the third term and ***A1*** for the fourth term.

$$1-x^2+x^4-x^6$$ ***AG***

[3 marks]

(ii) $x-\frac{1}{3}x^3+\frac{1}{5}x^5-\frac{1}{7}x^7\left(+C\right)$ ***A2***

[2 marks]

Note: Award ***A1A0*** for any three correct terms.

(b) (i) attempt to create binomial expansion the term in x^4 with $n=-\frac{1}{2}$ ***(M1)***

$$1+\binom{-\frac{1}{2}}{1}x^2+\binom{-\frac{1}{2}}{2}\left(x^2\right)^2+\ldots$$

$$1-\frac{1}{2}\left(-x^2\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(-x^2\right)^2$$ ***(A1)(A1)***

Note: Award ***(A1)*** for the first two terms and ***(A1)*** for the third term.

$$1+\frac{1}{2}x^2+\frac{3}{8}x^4$$ ***A1***

(ii) $x+\frac{1}{6}x^3+\frac{3}{40}x^5\left(+C\right)$ ***A1***

[5 marks]

continued…

Question 10 continued.

(c) $\arcsin x = \int \frac{\mathrm{d}x}{\sqrt{1-x^2}}$ (seen anywhere) ***(A1)***

attempt to substitute $x = \frac{1}{2}$ into their antiderivative ***(M1)***

$\left(\arcsin \frac{1}{2} \approx\right) \frac{1}{2} + \frac{1}{6}\left(\frac{1}{2}\right)^3 + \frac{3}{40}\left(\frac{1}{2}\right)^5$, $\frac{1}{2} + \frac{1}{48} + \frac{3}{1280}$ ***A1***

$k = 3$ (accept $\frac{25}{48} + \frac{3}{1280}$) ***A1***

[4 marks]

(d) $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$ (seen anywhere) ***(A1)***

equating their value of $\arcsin\left(\frac{1}{2}\right)$ to their expression from (c) ***(M1)***

$\frac{\pi}{6} \approx \frac{25}{48} + \frac{3}{1280}$, $\pi \approx 6 \times \left(\frac{25}{48} + \frac{3}{1280}\right)$

$\pi \approx \frac{25}{8} + \frac{18}{1280}$, $\frac{4000}{1280} + \frac{18}{1280}$ ***(A1)***

$\pi \approx \frac{2009}{640}$ (accept $p = 2009$) ***A1***

Note: Condone use of "$=$" instead of "$\approx$".

[4 marks]

Total [18 marks]

11. (a) attempt to multiply by conjugate ***M1***

$\frac{4\mathrm{i}}{1+\mathrm{i}\sqrt{3}} \times \frac{1-\mathrm{i}\sqrt{3}}{1-\mathrm{i}\sqrt{3}}$

$\frac{4\sqrt{3}+4\mathrm{i}}{4}$, $\mathrm{i}\left(1-\mathrm{i}\sqrt{3}\right)$ ***A1***

$w=\sqrt{3}+\mathrm{i}$ ***AG***

[2 marks]

(b) (i) attempt to find either $|w|$ or $\arg w$ (could be seen in (ii)) ***(M1)***

$|w|=\sqrt{1^2+3}$ OR $\arctan\left(\frac{1}{\sqrt{3}}\right)$ OR labelled diagram

$|w|=2$ ***A1***

[2 marks]

(ii) $\arg w=\frac{\pi}{6}$ (accept 30°) ***A1***

[1 mark]

(c) attempt to find a root using de Moivre's theorem with $n=\frac{1}{2}$ ***(M1)***

$\sqrt{2}\mathrm{e}^{\frac{\mathrm{i}\pi}{12}}$ ***A1***

attempt to find second root by adding or subtracting π from *their* argument ***(M1)***

$\sqrt{2}\mathrm{e}^{\frac{-11\pi\mathrm{i}}{12}}$ ***A1***

[4 marks]

continued…

Question 11 continued.

(d) (i) **METHOD 1**

attempt to expand and equate to $\sqrt{3}+\mathrm{i}$ ***(M1)***

$a^2 - b^2 + 2ab\mathrm{i} = \sqrt{3}+\mathrm{i}$

at least one correct equation ***A1***

$a^2 - b^2 = \sqrt{3}$ **OR** $2ab = 1$

substituting $b = \frac{1}{2a}$ ***(M1)***

$a^2 - \left(\frac{1}{2a}\right)^2 = \sqrt{3}$

$4a^4 - 4\sqrt{3}a^2 - 1 = 0$ **OR** $a^4 - \sqrt{3}a^2 - \frac{1}{4} = 0$ ***A1***

attempt to solve quadratic in a^2 ***M1***

$a^2 = \frac{4\sqrt{3} \pm \sqrt{48+16}}{8}$ OR $\frac{\sqrt{3}}{2} \pm 1$

since $a^2 > 0$ (accept $a > 0$) ***R1***

$a^2 = \frac{\sqrt{3}}{2} + 1$ ***AG***

continued…

Question 11 continued.

METHOD 2

$\sqrt{w} = a + bi$ ***(A1)***

$a + bi = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$ ***A1***

attempt to equate $\text{Re}\left(\sqrt{w}\right)$ ***M1***

$a = \sqrt{2}\cos\frac{\pi}{12}$

$a^2 = 2\cos^2\frac{\pi}{12}$ ***A1***

attempt to use double angle formula ***(M1)***

$\cos 2\alpha = 2\cos^2\alpha - 1$

$a^2 = 2\left(\frac{\cos\frac{\pi}{6} + 1}{2}\right) = \left(\cos\frac{\pi}{6} + 1\right)$ ***A1***

$\left(a^2 =\right)\frac{\sqrt{3}}{2} + 1$ ***AG***

[6 marks]

(ii) attempt to substitute a^2 into $b^2 = a^2 - \sqrt{3}$ OR $b^2 = \frac{1}{4a^2}$ ***(M1)***

$\left(\frac{\sqrt{3}}{2} + 1\right) - \sqrt{3}$, $\frac{1}{4\left(\frac{\sqrt{3}}{2} + 1\right)}$

$b^2 = 1 - \frac{\sqrt{3}}{2}$ ***A1***

Note: Accept the unrationalized answer of $\frac{1}{2\sqrt{3} + 4}$.

[2 marks]

continued…

Question 11 continued.

(e) attempt to set up an equation using an argument of $\frac{\pi}{12}$ ***(M1)***

$\tan\left(\frac{\pi}{12}\right) = \frac{b}{a} = \sqrt{\frac{b^2}{a^2}}$ ***(A1)***

substitution of the given a and their b ***M1***

$\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}}$ **OR** $\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}$ ***A1***

$\sqrt{\frac{\left(2-\sqrt{3}\right)^2}{2^2-3}}$

$\tan\left(\frac{\pi}{12}\right) = 2-\sqrt{3}$ **,** accept $p = 2,\ q = -1$ ***A1***

[5 marks]

Total [22 marks]