Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 1

10 November 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

8825 – 7106

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The 1st and 5th terms of an arithmetic sequence are 36 and 12 respectively.

(a) Find the 13th term of this arithmetic sequence. [4]

The sum of the first n terms of this arithmetic sequence is zero.

(b) Find the value of n. [2]

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2. [Maximum mark: 6]

(a) Given that $x > 7$, show that $\frac{x}{x^2 - 8x + 7} \times \frac{x^2 - 1}{x + 1} \equiv \frac{x}{x - 7}$. [2]

(b) Hence, or otherwise, solve $\log_2[x(x^2 - 1)] - 1 = \log_2[(x^2 - 8x + 7)(x + 1)]$. [4]

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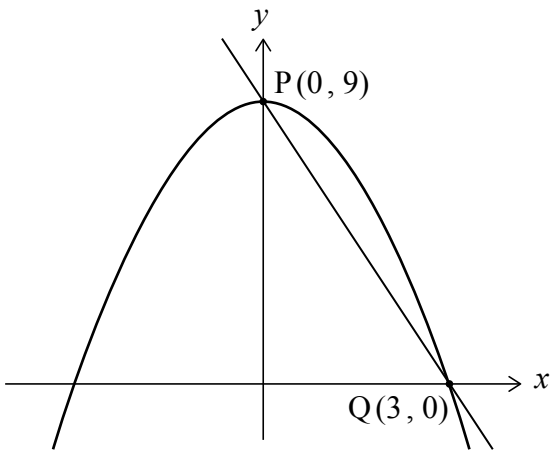
3. [Maximum mark: 7]

A line with equation $y = -3x + 9$ intersects the axes at the points $\mathrm{P}(0, 9)$ and $\mathrm{Q}(3, 0)$.

A parabola of the form $y = ax^2 + c$, where $a, c \in \mathbb{Z}$, also passes through the points P and Q.

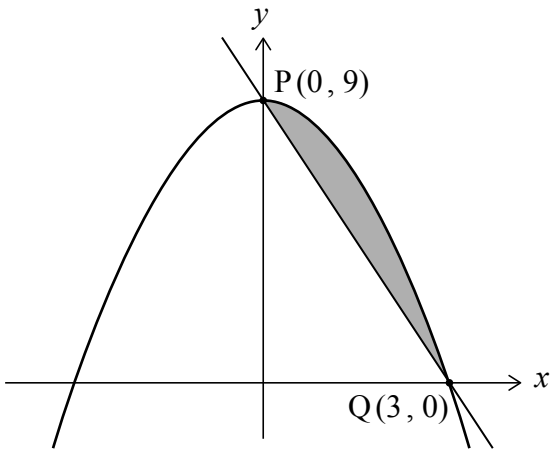
This is shown in the following diagram.



(a) (i) Write down the value of c.

(ii) Find the value of a. [3]

The region enclosed by the line and the parabola is shaded in the following diagram.



(b) Find the area of the shaded region. [4]

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(Question 3 continued)

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4. [Maximum mark: 8]

For $x \in \mathbb{R}$, the functions f and g are defined by

$$f(x) = -x^2 + 4x + p \text{ and}$$

$$g(x) = x^2 + qx - 1,$$

where p and q are non-zero constants.

The graph of $y = f(x)$ has a vertex at A and the graph of $y = g(x)$ has a vertex at B.

Points A and B both lie on the line $y = 2x - 1$.

(a) Show that $p = -1$. [3]

(b) Find the value of q. [5]

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5. [Maximum mark: 7]

The first four terms of the Maclaurin series expansion of $(1-x)^{-4}$ are

$$1 + ax + bx^2 + 20x^3,\ a, b \in \mathbb{Z}^+.$$

(a) (i) Show that $a = 4$.

(ii) Find the value of b. [5]

A car was purchased four years ago. The car depreciated in value by 10% each year. The value of the car today is $\$1000$.

(b) Using the results of part (a), estimate the value of the car four years ago. [2]

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6. [Maximum mark: 6]

A linear system of equations is given by

$$x - z = 4$$

$$2x + 2y + z = 14$$

$$x + 2y + \alpha z = \beta$$

Find the value of α and the value of β, where $\alpha, \beta \in \mathbb{Z}$, for which the system has an infinite number of solutions.

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7. [Maximum mark: 8]

Consider the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 \cos^2 x \, , \; y \neq 0$$

The solutions to this differential equation can be expressed in the form

$$y = \frac{a}{2x + \sin 2x + b} \, ,$$

where a and b are constants such that $a \in \mathbb{Z}$ and $b \in \mathbb{R}$.

(a) By solving the differential equation, find the value of a. [6]

(b) Given that $y = 1$ when $x = \frac{\pi}{2}$, find the value of b. [2]

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8. [Maximum mark: 8]

(a) Show that $\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta$. [4]

(b) Hence, describe a sequence of transformations that maps the graph of $y = \sin\theta$ onto the graph of $y = 6\sin\left(\theta + \frac{\pi}{6}\right) - 8\sin^3\left(\theta + \frac{\pi}{6}\right)$. [4]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

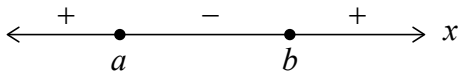
9. [Maximum mark: 14]

The function f has a derivative given by $f'(x) = 3x^2 + 12x - 15$.

The graph of $y = f(x)$ has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

(a) Find the value of a and the value of b. [3]

The following diagram shows three intervals along the x-axis defined by a and b. The sign of the **first derivative** of f is shown in each interval.

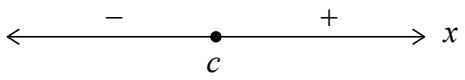


(b) State, with a reason, whether there is a local maximum point or a local minimum point on the graph of $y = f(x)$ at $x = a$. [2]

The second derivative $f''(x)$ is zero at $x = c$.

(c) Find the value of c. [3]

The following diagram shows two intervals along the x-axis defined by c. The sign of the **second derivative** is shown in each interval.



(d) State, with a reason, whether there is a point of inflexion on the graph of $y = f(x)$ at $x = c$. [2]

(e) Given that $f(-2) = 36$, find $f(x)$. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 18]

Consider the expression $\frac{1}{1+x^2}$, where $|x| < 1$.

(a) (i) Show that the first four terms in the binomial expansion of $\frac{1}{1+x^2}$ are $1 - x^2 + x^4 - x^6$. [3]

(ii) Hence, find an approximation for $\int \frac{1}{1+x^2}\,\mathrm{d}x$ up to and including the term in x^7. [2]

Now consider the expression $\frac{1}{\sqrt{1-x^2}}$, where $|x| < 1$.

(b) (i) Expand $\frac{1}{\sqrt{1-x^2}}$ up to and including the term in x^4.

(ii) Hence, find a polynomial expression for $\int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x$ up to and including the term in x^5. [5]

The expression $\frac{25}{48} + \frac{k}{1280}$, where $k \in \mathbb{Z}^+$, can be used to approximate $\arcsin\frac{1}{2}$.

(c) Use the result from part (b)(ii) to find the value of k. [4]

(d) Hence, find an approximation for π in the form $\frac{p}{640}$ where $p \in \mathbb{Z}^+$. [4]

Do **not** write solutions on this page.

11. [Maximum mark: 22]

Consider the complex number $w = \dfrac{4\mathrm{i}}{1+\mathrm{i}\sqrt{3}}$.

(a) Show that $w = \sqrt{3} + \mathrm{i}$. [2]

(b) Find

(i) $\left|w\right|$;

(ii) $\arg w$. [3]

(c) Use De Moivre's theorem to find the two square roots of w. Give your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [4]

The complex number w can be expressed in the form $(a + b\mathrm{i})^2$, where $a, b \in \mathbb{R}^+$.

(d) (i) Show that $a^2 = \dfrac{\sqrt{3}}{2} + 1$.

(ii) Find the value of b^2. [8]

(e) Hence, express $\tan\dfrac{\pi}{12}$ in the form $p + q\sqrt{3}$, where $p, q \in \mathbb{Z}$. [5]

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