

# Markscheme

**May 2025**

**Mathematics: applications and  
interpretation**

**Higher level**

**Paper 3**

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### Instructions to Examiners

#### Abbreviations

- M** Marks awarded for attempting to use a correct **Method**.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- R** Marks awarded for clear **Reasoning**.
- AG** Answer given in the question and so no marks are awarded.
- FT** Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

#### Using the markscheme

##### 1 General

Award marks using the annotations as noted in the markscheme *eg M1, A2*.

##### 2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, *e.g. M1A1*, this usually means **M1** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies **A3, M2** etc., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the **AG** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award **FT** marks as appropriate but do not award the final **A1** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	No. Last part in question.	Award <b>A1</b> for the final mark (condone the incorrect further working)
2.	$\frac{35}{72}$	0.468111... (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award <b>A0</b> for the final mark (and full <b>FT</b> is available in subsequent parts)

### 3 Implied marks

Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

### 4 Follow through marks (only applied after an error is made)

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then **FT** marks should be awarded for *their* correct answer, even when working is not present.

**For example:** following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is **(M1)A1**, it is possible to award full marks for *their* correct answer, **without working being seen**. For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (e.g. probability greater than 1,  $\sin \theta = 1.5$ , non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any **FT** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these **FT** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

## 5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread and do not award the first mark, even if this is an **M** mark, but award all others as appropriate.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (e.g. probability greater than 1,  $\sin \theta = 1.5$ , non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- **MR** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

## 6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for parts of questions are indicated by **EITHER . . . OR**.

## 7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, **M** marks and intermediate **A** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

## 8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come “*from the use of 3 sf values*”.

**Simplification of final answers:** Candidates are advised to give final answers using good mathematical form. In general, for an **A** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example,  $\sqrt{\frac{25}{4}}$  should be written as  $\frac{5}{2}$ . An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example,  $\frac{10}{4}$  may be left in this form or written as  $\frac{5}{2}$ .

However,  $\frac{10}{5}$  should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g.  $4e^{2x} \times e^{3x}$  should be simplified to  $4e^{5x}$ , and  $4e^{2x} \times e^{3x} - e^{4x} \times e^x$  should be simplified to  $3e^{5x}$ . Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so  $x(x+1)$  and  $x^2 + x$  are both acceptable.

**Please note:** intermediate **A** marks do NOT need to be simplified.

## 9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

## 10. Presentation of candidate work

**Crossed out work:** If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

**More than one solution:** Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) (i)  $y = 5$

**A1A1**

**Note:** Award **A1** for  $y =$  a constant, **A1** for 5.

**[2 marks]**

(ii) equating the correct lines

**(M1)**

$5 = 15 - 2x$  OR sketch graph

$x = 5$   $y = 5$  OR (5,5)

**A1**

**[2 marks]**

(b) station B

**A1**

**[1 mark]**

(c) nearest neighbour (interpolation)

**A1**

**[1 mark]**

(d) **EITHER**

because the school is quite close to the other two stations also and their readings are much higher.

**R1**

**OR**

the reading of 49 is only an average, and hence likely that readings are frequently over 50 at the school.

**R1**

**Note:** Ignore any extra invalid reasons.

**[1 mark]**

(e) (i) attempt to find a midpoint

**(M1)**

$$\left( \frac{5+8}{2}, \frac{6+9}{2} \right)$$

(6.5, 7.5)

**A1**

**[2 marks]**

*continued...*

Question 1 continued

(ii) gradient is  $\frac{9-6}{8-5} = 1$  **A1**

gradient of perpendicular bisector is  $-1$  **A1**

substituting their gradient and their mid-point into the equation of a line **M1**

$$y - 7.5 = -1(x - 6.5) \text{ OR } 7.5 = -1 \times 6.5 + c$$

$$y = 14 - x$$
**AG**

**Note:** : If **A1A1M1** have been awarded but there is then incorrect work after the final **M1** such that the **AG** does not follow, do not award the final **M1**.

**[3 marks]**

(iii) attempt to find where this line meets the other two lines **(M1)**

$(9,5)$  **A1**

$$14 - x = 15 - 2x$$

$(1,13)$  **A1**

**Note:** These two marks are un-implicit due to the “hence” in the question. At least one needs to be seen for the **(M1)** to be awarded.

$$\text{Area} = \frac{1}{2} \times 4 \times 8$$
**(M1)**

$$= 16(\text{km}^2)$$
**A1**

**Note:** Award **(M1)** for any valid method which is completed (for example cosine rule, followed by area of triangle formula or using one of the other sides as base and correctly finding the perpendicular height) which leads to an answer, even if the answer is not correct due to a calculation error.

(the side lengths of the triangle are 4, 11.3, 8.94 and the angles  $45^\circ$ ,  $18.4^\circ$ ,  $116.6^\circ$ )

**Note:** Award **(M0)A0** for an unsupported answer of 16.

**[5 marks]**  
continued...

Question 1 continued

(f) attempt to substitute the values into given formula

**M1**

$$W = \frac{132 \times 13.7 + 49 \times 16 + 125 \times 6.9}{16 + 13.7 + 6.9}$$

**A1**

$$W = 94.4(94.3961\dots)$$

**AG**

**[2 marks]**

(g)  $H_0 : \mu = 94.4, H_1 : \mu > 94.4$

**A1A1**

**Note:** Allow the hypotheses in words, but award **A0A0** if 'mean' is not seen and, at most **A1A0** if it is not clear that the **population** mean is being tested

Accept  $\mu \leq 94.4$  for the null hypothesis.

use of *t*-test

**(M1)**

$$p\text{-value} = 0.0655 (0.0655348\dots)$$

**A1**

$$0.0655 < 0.10$$

**R1**

(hence reject the null hypothesis or accept the alternative hypothesis)

(there is significant evidence that) the (mean) AQI value (at the school) is greater than 94.4

**A1**

**Note:** Do not award **R0A1**.

Follow through from an incorrect *p*-value to award the last two marks,

Award **A0** for the conclusion if it is not in context or if the null hypothesis is incorrect, though condone the errors in language detailed in the first note above.

**[6 marks]**

(h)  $P(X < 50) = 0.00273(0.00272574\dots)$

**(A1)**

$$P(X < 50) \times 365$$

**(M1)**

$$= 0.994895\dots$$

$$= 1$$

**A1**

**[3 marks]**

**Total [ 28 marks]**

2. (a) 310 million is the maximum no of EVs **A1**  
**OR**  
the carrying capacity is 310 million **A1**  
**OR**  
the long term behaviour of the system approaches 310 million **A1**

**Note:** 'Million' needs to be seen to award the **A1**.

**[1 mark]**

- (b) (i) correct substitution  $0.2 = \frac{310}{1 + Ce^{-k}}$  **A1**  
correct rearrangement which leads to correct final answer **A1**  
eg  $1 + Ce^{-k} = 1550$  OR  $Ce^{-k} = 1549$  OR  $0.2Ce^{-k} = 309.8$   
 $C = 1549e^k$  **AG**

**[2 marks]**

- (ii) correct substitution  $3.1 = \frac{310}{1 + Ce^{-7k}}$  **A1**  
correct first stage of rearrangement **(A1)**  
eg  $1 + Ce^{-7k} = 100$  OR  $3.1 + 3.1Ce^{-7k} = 310$   
 $C = 99e^{7k}$  **A1**

**Note:** Allow any correct expression eg  $C = \frac{99}{e^{-7k}}$  or  $C = \frac{306.9}{3.1e^{-7k}}$ .

**[3 marks]**

*continued...*

Question 2 continued

(c) valid method for finding either variable (M1)

eg graphically or elimination of  $C$ ,  $e^{6k} = \frac{1549}{99}$

(i)  $k = 0.458$  (0.458374...) A1

**Note:** Award at most **M1A0** if values other than at  $t = 1$  or  $t = 7$  are used.

[2 marks]

(ii)  $C = 2450$  (2449.74...) A1

**Note:** Do not award **A1FT** on a wrong value for  $k$  if the method used is not clearly shown.

**Note:** Accept  $C = 2440$  (2443.33...) from substitution of 3 sf value for  $k$  in  $C = 99e^{7k}$

**Note:** Award **A0** if values other than at  $t = 1$  or  $t = 7$  are used.

[1 mark]

$$\left( N = \frac{310}{1 + 2449.74...e^{-0.458374...t}} \right)$$

(d) substitution of  $t = 4$  into their logistic function

$$N = \frac{310}{1 + 2449.74...e^{-0.458374... \times 4}} \quad (M1)$$

$$a = 0.789606...$$

**Note:** If using 3 sf values for  $k$  and  $C$ :  $a = 0.788346...$  . If using  $C = 2440$  then  $a = 0.792$  .

$$a = 0.8 \quad A1$$

**Note:** The answer must be given correct to 1 decimal place

[2 marks]

(e) calculation of a squared residual (M1)

$$0^2 + 0.1^2 + 0.2^2 + 0.1^2 + 0.1^2$$

$$\text{sum of square residuals} = 0.07 \quad A1$$

[2 marks]

continued...

Question 2 continued

- (f) substitution of their values into the error function **M1**

**Note:** For the **M1** allow any integer value of  $n$  being substituted, with their sum of square residuals.

$$\left( E = \sqrt{\frac{0.07}{5}} \right) = 0.118 \text{ (0.118321...)} \quad \mathbf{A1}$$

$$0.118 < 0.25 \text{ (so can use the model)} \quad \mathbf{R1}$$

**[3 marks]**

- (g) number in 2035 is given by value of  $N$  when  $t = 20$  **(A1)**  
 247 million (246.873...) **A1**

**Note:** Accept 246 million from using 3 sf values for  $k$  and  $C$ .  
 Award **(A1)A0** for 247 or 246 without 'million'.  
 If using  $t = 19$  award **(A0)A1** for 220 or 221 million. If using  $t = 21$  award **(A0)A1** for 267 million.

**[2 marks]**

- (h) multiplication of their **model** by one of 0.2 or 0.1 **M1**

**Note:** For the **M1** ignore any extra incorrect multipliers.

number of public charging points needed in year  $t$  is

$$\frac{310}{1 + 2449.74...e^{-0.458374...t}} \times 0.2 \times 0.1 \left( = \frac{6.2}{1 + 2449.74...e^{-0.458374...t}} \right) \text{ (million) OR}$$

$$\frac{310,000,000}{1 + 2449.74...e^{-0.458374...t}} \times 0.2 \times 0.1 \left( = \frac{6,200,000}{1 + 2449.74...e^{-0.458374...t}} \right) \quad \mathbf{A1}$$

**[2 marks]**

- (i)  $\left( \frac{0.54 - 0.22}{2} \right) = 0.16 \text{ (million) (160,000)} \quad \mathbf{A1}$

**[1 mark]**

continued...

Question 2 continued

(j) attempt to express number of charging points at  $t$  as a linear function **(M1)**

linear function passes through  $(5, 0.22)$  or  $(7, 0.54)$  **(A1)**

$0.22 + 0.16(t - 5)$  OR  $0.54 + 0.16(t - 7)$  OR  $0.16t - 0.58$  OR  $160,000t - 580,000$  **A1**

**Note:** Award at most **(M1)(A1)A1** for an expression that has the variable in years, for example  $0.22 + 0.16(x - 2020)$ .

attempt to solve their  $\frac{6.2}{1 + 2449.74...e^{-0.458374...t}} > 0.22 + 0.16(t - 5)$  **M1**

**Note:** Accept = in place of >.

$t = 15.1$  (15.1492...) **(A1)**

**Note:** Accept 15.1665... from use of 3 sf values for  $k$  and  $C$ .

Year is 2031 **A1**

**[6 marks]**

**Total [27 marks]**

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