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Diploma Programme
Programme du diplôme
Programa del Diploma

# Mathematics: applications and interpretation
# Higher level
# Paper 3

21 May 2025

**Zone A** afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 15 minutes

## Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.



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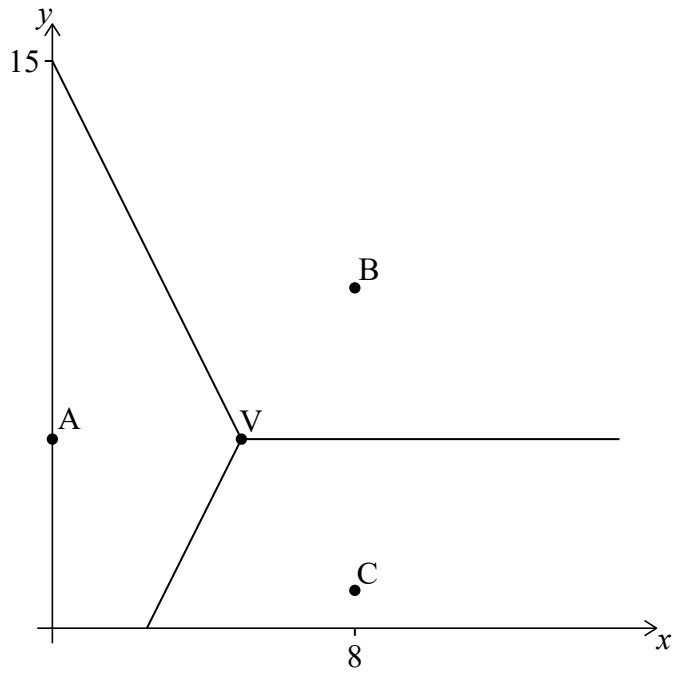
Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

**1.** [Maximum mark: 28]

**This question considers how the assessment of the Air Quality Index (AQI) for a school depends on the method chosen by the person doing the assessing.**

Air quality for a district is measured at three monitoring stations. The positions of these stations on a coordinate system with units in kilometres are $\mathrm{A}(0, 5)$, $\mathrm{B}(8, 9)$ and $\mathrm{C}(8, 1)$.

A Voronoi diagram is constructed with the three stations as sites.




(a) (i) Find the equation of the perpendicular bisector of $[\mathrm{BC}]$. [2]

(ii) Given that the equation of the perpendicular bisector of $[\mathrm{AB}]$ is $y = 15 - 2x$, find the coordinates of vertex $\mathrm{V}$. [2]

**(This question continues on the following page)**

**(Question 1 continued)**

A school, $S$, is situated in the district at the point with coordinates $(5, 6)$.

(b) State which station is closest to the school. [1]

The principal of the school is concerned about the air quality in the area. Air quality is measured by the Air Quality Index (AQI). In this district, values less than $50$ are taken to indicate good air quality.

The principal contacts the local environmental agency requesting an AQI value for her school. They tell her the mean AQI reading from the closest station to the school.

(c) Write down the type of interpolation being used by the environmental agency. [1]

The principal obtains the mean AQI value from each of the three stations.

| Station | Mean AQI value |
|---|---|
| A | 132 |
| B | 49 |
| C | 125 |

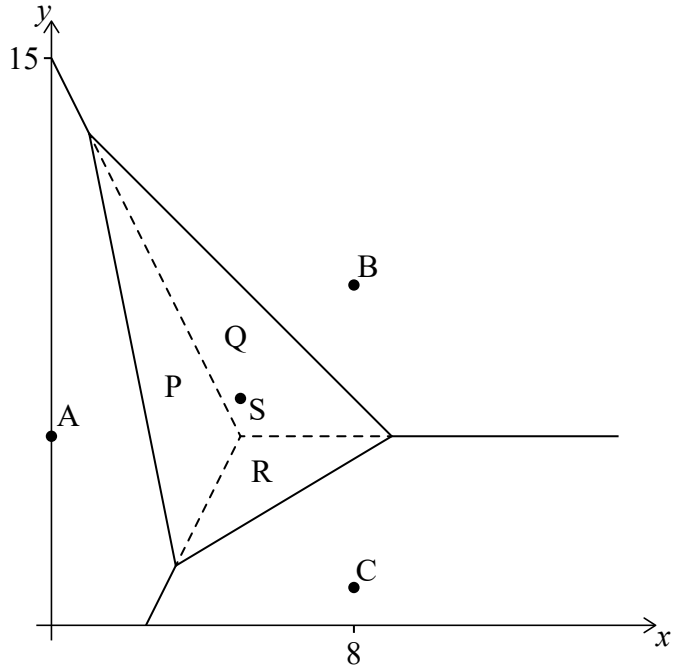
(d) Explain why the principal might not accept that the air quality around the school can be classed as “good”. [1]

**(This question continues on the following page)**

**(Question 1 continued)**

The principal decides to obtain an expected value for the AQI at the school that uses all the available information. To do this, she uses an alternative method: the natural neighbour algorithm. This algorithm has two stages.

The first stage is to create a new Voronoi diagram with the school as an extra site. This is shown in the following diagram with the edges of the previous diagram shown by dashed lines.




The second stage is to estimate the AQI value at the school, $W$, by using the formula

$$W = \frac{w_A a_A + w_B a_B + w_C a_C}{T}.$$

In the formula, $a_A$ is the area within the new cell that has been taken from the cell surrounding site $\mathrm{A}$, shown as region $\mathrm{P}$ on the diagram, and $w_A$ is the mean AQI value from site $\mathrm{A}$. This is given as $132$ in the table of mean AQI values above. Similarly for sites $\mathrm{B}$ and $\mathrm{C}$. $T$ is the total area of the new cell around $\mathrm{S}$.

(e) (i) Find the mid-point of $[\mathrm{SB}]$. [2]

(ii) Show that the equation of the perpendicular bisector of $[\mathrm{SB}]$ is $y = 14 - x$. [3]

(iii) Hence find $a_B$ (the area of region $\mathrm{Q}$ on the diagram). [5]

**(This question continues on the following page)**

**(Question 1 continued)**

The areas of regions P and R are $a_A = 13.7\,\text{km}^2$ and $a_C = 6.9\,\text{km}^2$ respectively.

(f) Use the natural neighbour algorithm to show that an estimate for the expected AQI value at the school, $W$, is $94.4$. [2]

The principal is still concerned that this method is underestimating the AQI value at the school, as the school is situated close to a busy traffic intersection. She decides to take her own readings $(x)$ over a period of $60$ days. Her results are summarized as

$$\bar{x} = 97.8,\ s_{n-1} = 17.2,\ n = 60.$$

The principal assumes that the daily AQI values at the school can be modelled by a normal distribution and that each value is independent of any other value.

(g) Test at the $10\%$ significance level the hypothesis that the mean AQI value at the school is greater than $94.4$. State clearly the null and alternative hypotheses and the conclusion of the test in context. [6]

The principal now assumes that the distribution of AQI values at the school follows a normal distribution with mean $97.8$ and standard deviation $17.2$.

(h) Use this model to find the expected number of days per year (correct to the nearest day) on which the AQI value can be classed as “good”. [3]

**2.** [Maximum mark: 27]

**George is researching the growth in the number of electric vehicles (EVs) in the European Union in order to investigate some of the difficulties that might arise if the target of banning sales of all petrol and diesel cars in 2035 is to be met.**

George begins his research by predicting how many electric vehicles (EVs) will be in the European Union in 2035.

The number of EVs in the European Union, $N$, measured in millions, is shown in the following table. The time $t$ is measured in years from the **beginning** of 2016 where $t \in \mathbb{R}$.

| End of year | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 | 2022 |
|---|---|---|---|---|---|---|---|
| $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $N$ | 0.2 | 0.3 | 0.4 | 0.6 | 1.1 | 1.9 | 3.1 |

George models this data set using the logistic function $N = \frac{310}{1 + C\mathrm{e}^{-kt}}$, where $C, k \in \mathbb{R}^+$.

(a) In context, explain the significance of $310$ in George's logistic function. [1]

(b) (i) Use the value of $N$ at $t = 1$ to show that $C = 1549\mathrm{e}^{k}$. [2]

(ii) Use the value of $N$ at $t = 7$ to find a second expression for $C$. [3]

(c) Use your answers to part (b) to find a value for

(i) $k$. [2]

(ii) $C$. [1]

George uses his model to predict values of $N$ at the end of the years 2017 to 2021. These values are shown in the following table, correct to one decimal place.

| End of year | 2017 | 2018 | 2019 | 2020 | 2021 |
|---|---|---|---|---|---|
| $t$ | 2 | 3 | 4 | 5 | 6 |
| $N$ | 0.3 | 0.4 | 0.6 | 1.1 | 1.9 |
| Predicted values | 0.3 | 0.5 | $a$ | 1.2 | 2.0 |

(d) Calculate the value of $a$ correct to one decimal place. [2]

(e) Using the value of $a$ correct to one decimal place, find the sum of square residuals, $SS_{res}$, when using this model to predict the values of $N$. [2]

**(This question continues on the following page)**

**(Question 2 continued)**

As a measure of how well the model fits the data, George uses the error function, $E$,

where $E = \sqrt{\frac{SS_{res}}{n}}$, and $n$ is the number of predictions made using the model.

George decides he will use this model if $E$ is less than $0.25$.

(f) By finding the value of $E$ for George's model, show that George will decide that the model can be used. [3]

(g) Use the model to predict the number of EVs in the European Union at the end of 2035. [2]

One of the main difficulties in meeting the European Union's target is the provision of sufficient public charging points.

The European Union estimates that $80\%$ of car owners will be able to charge their cars at home and the remaining $20\%$ will require public charging points. It is planned to have one public charging point for every $10$ cars that require them.

(h) Use George's model to find an expression for the total number of public charging points required at time $t$. [2]

At the end of 2020 there were $0.22$ million public charging points in the European Union and at the end of 2022 there were $0.54$ million.

(i) Find the average number of public charging points installed per year during this period. [1]

George assumes that the number of public charging points will continue to increase linearly, at the same rate, until 2035.

(j) Determine the value of $t$, where $t \geq 5$, at which the number of charging points will first be insufficient to meet demand, and hence the year in which this occurs. [6]