Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 3

21 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 15 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

2225 – 7313

9 pages

Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 28]

This question is about comparing the academic performance of two schools.

At age 18, all students in school A and school B take the same final exam. Augustin is studying the results in these schools.

Augustin chooses to take a representative sample of size six from each school. For each student in the sample, he will conduct an interview.

(a) (i) State one advantage of increasing the sample size. [1]

(ii) State one disadvantage of increasing the sample size. [1]

The data in **Table 1** shows the results of the final exam as a percentage for each of the six students, sampled from school A.

Table 1

Candidate number	Final exam
10001	63.5
10003	52.5
10012	50.7
10045	42.8
10073	44.7
10090	56.1

The mean result for the sample from school A is 51.7 to three significant figures.

(b) Find the value of s_{n-1} for the sample from school A. [2]

(This question continues on the following page)

(Question 1 continued)

The value of s_{n-1} for the sample from school B is 7.66 to three significant figures.

Augustin makes the following claim:

"The spread of results in school A must be less than the spread of results in school B."

(c) Make one criticism of Augustin's claim. [1]

The examination board claims that the final exam results in each school are approximately normally distributed. You may assume that this claim is correct.

Augustin decides to use a pooled t-test to compare the mean results of school A and of school B.

(d) (i) State the condition regarding population variances required to use a pooled t-test. [1]

(ii) State whether Augustin should use a pooled t-test in this case. Justify your answer. [2]

Prior to collecting data, Augustin believed that the mean result of school B was higher than that of school A. From his data he finds that the mean final exam result for the sample from school B is exactly 60.

(e) (i) State appropriate null and alternative hypotheses for the pooled t-test. [2]

(ii) Find the *p*-value. [2]

(iii) Given that the test is carried out at the 5% significance level, state the appropriate conclusion in context. Justify your answer. [2]

All students in the two schools took the same entry exam at age 11.

Augustin wants to determine if there is evidence of any correlation between the entry exam result and final exam result. For the 12 students in the sample, Augustin collects their entry exam results.

The Pearson's product moment correlation coefficient between the results for the entry exam and the final exam is $r = 0.876$ to three significant figures. The critical value of r at the 5% significance level is 0.576.

(f) (i) Assuming all requirements are met, perform a test at the 5% significance level.

State the hypotheses and justify your conclusion. [4]

(ii) If the requirements are not met, state an alternative test. [1]

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(Question 1 continued)

The examination board uses a model to make a prediction of a student's final exam result $(\hat{y})$ based on their entry exam result (x).

The model used is:

$$\hat{y} = 0.37x + 37.6$$

(g) Give, in context, an interpretation of the gradient 0.37 in the model. [1]

For each student, Augustin uses the value $y - \hat{y}$ rounded to one decimal place to measure the extent to which a school helped to improve a student's results. He calls this "school value added". This is shown in **Table 2**.

Table 2

Candidate number	School	Entry exam (x)	Final exam (y)	School value added $(y - \hat{y})$
10001	A	53.2	63.5	6.2
10003	A	38.7	52.5	q
10012	A	32.0	50.7	1.3
10045	A	20.0	42.8	–2.2
10073	A	23.2	44.7	–1.5
10090	A	42.4	56.1	2.8
20004	B	72.0	60.8	–3.4
20008	B	69.5	60.3	–3.0
20023	B	52.3	47.1	–9.9
20044	B	84.1	69.9	1.2
20061	B	74.7	64.6	–0.6
20083	B	66.9	57.3	–5.1

(h) (i) Show that the value of q in **Table 2** is 0.6. [2]

(ii) Assuming that the appropriate requirements are met, use a pooled t-test at a 5% significance level to determine if the mean "school value added" is higher in school A than in school B. Write down your null and alternative hypotheses and justify your conclusion. [4]

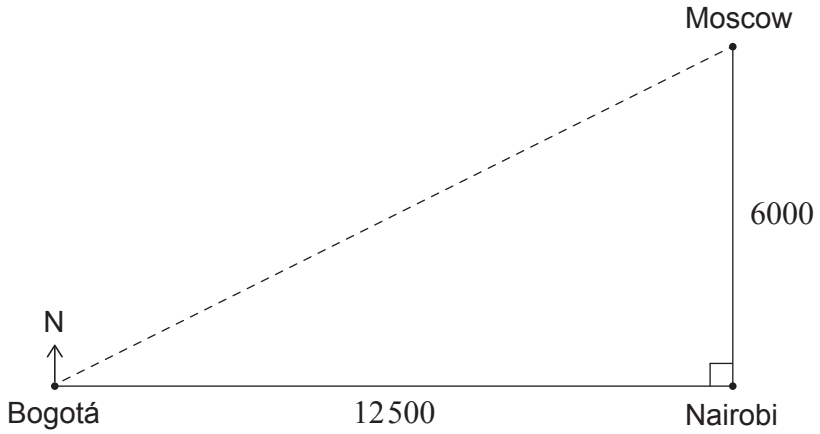
(i) Using Augustin's results, explain how each school could claim they are performing better than the other school. [2]

2. [Maximum mark: 27]

The following question compares the distance and direction between cities on a flat surface to the distance and direction between cities on a sphere.

Consider a model where the cities of Bogotá, Moscow, and Nairobi lie on a flat surface. In this model, Nairobi is $6000\,\text{km}$ due south of Moscow and Bogotá is $12\,500\,\text{km}$ due west of Nairobi, as shown in the following diagram.

diagram not to scale



(a) (i) Find the distance from Bogotá to Moscow. [2]

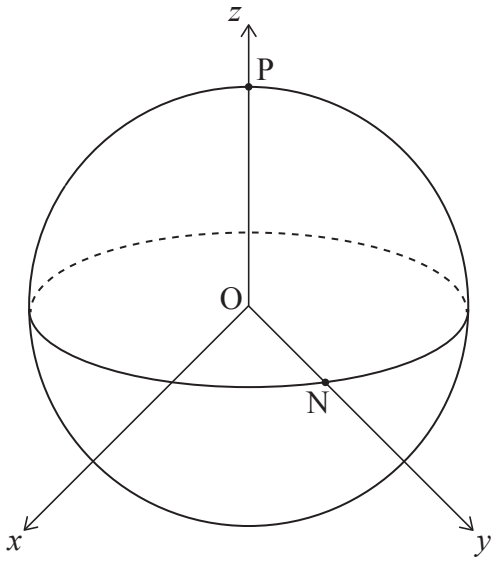
(ii) Find the bearing of Moscow from Bogotá. Give your answer in degrees. [3]

(This question continues on the following page)

(Question 2 continued)

In reality, these three cities lie on the curved surface of the Earth which will change the distances and directions found in part (a).

Now consider a curved model using a coordinate system (x, y, z) with its origin, O, at the centre of the Earth. The units of this system are thousands of kilometres and the Earth is modelled as a sphere with radius $6000\,\mathrm{km}$. The North Pole, P, lies on the z-axis, and Nairobi, N, is modelled as being on the equator and lying on the y-axis.



P has position vector $\overrightarrow{\mathrm{OP}} = \boldsymbol{p} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$ and N has position vector $\overrightarrow{\mathrm{ON}} = \boldsymbol{n} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$.

(b) (i) Use the scalar product to find the angle between $\boldsymbol{p}$ and $\boldsymbol{n}$. [2]

(ii) Show that the distance between P and N along the arc from P to N is $3000\pi\,\mathrm{km}$. [2]

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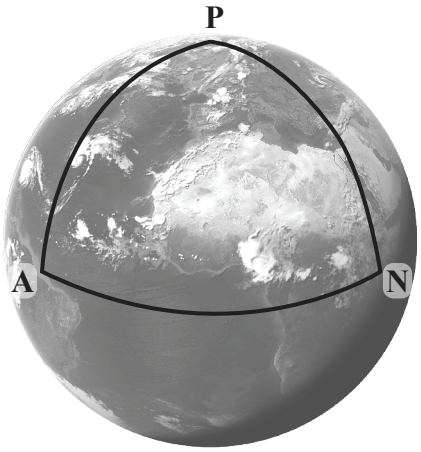
(Question 2 continued)

Point A, which is also on the equator, has position vector $\boldsymbol{a} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ as shown in the following diagram.



P, N and A, and the arcs connecting them, form a spherical triangle.

The angle at vertex A is defined as the angle between the vectors $\boldsymbol{a} \times \boldsymbol{p}$ and $\boldsymbol{a} \times \boldsymbol{n}$.

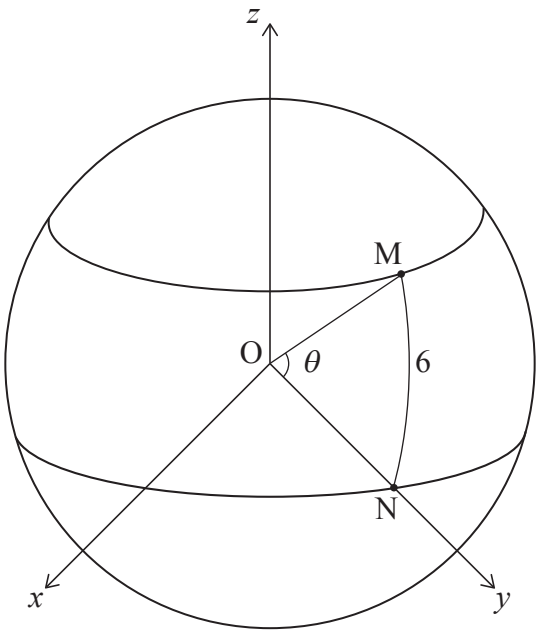
(c) (i) Find the vector $\boldsymbol{a} \times \boldsymbol{p}$. [2]

(ii) Show that the angle at vertex A in the spherical triangle is $90°$. [3]

(This question continues on the following page)

(Question 2 continued)

Moscow, M, has position vector $\overrightarrow{\mathrm{OM}} = \boldsymbol{m} = \begin{pmatrix} 0 \\ 6\cos\theta \\ 6\sin\theta \end{pmatrix}$, as shown in the following diagram.



The shortest distance between two points on the sphere lies along an arc of a circle on the sphere with centre O. In this model the shortest distance from Moscow to Nairobi is $6000\,\mathrm{km}$.

(d) Show that $\theta = 57.3°$, correct to three significant figures. [2]

Bogotá, B, is west of Nairobi and has position vector $\overrightarrow{\mathrm{OB}} = \boldsymbol{b} = \begin{pmatrix} 6\sin 120^\circ \\ 6\cos 120^\circ \\ 0 \end{pmatrix}$.

(e) Find the shortest distance from Bogotá to Moscow on the sphere. [5]

The bearing from B to M is defined as the angle at vertex B in the spherical triangle containing B, M and P. It is given that $\boldsymbol{b} \times \boldsymbol{p} = \begin{pmatrix} 36\cos 120^\circ \\ -36\sin 120^\circ \\ 0 \end{pmatrix}$.

(f) Using the method from part (c), find the bearing from Bogotá to Moscow. [6]

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References:

2(c) Google Maps/Google Earth. Data from Data SIO, NOAA, U.S. Navy, NGA, GEBCOLandsat / Copernicus, IBCAO, U.S. Geological Survey, PGC/NASA. Imagery from 14/12/2015. Image available at: https://earth.google.com/web/@12.01529518,-18.56070747,-158.39383184a,23597813.93249989d,30.00008083y,359.99981502h,0t,0r/data=CgRCAggBOgMKATBCAggASg0I_____ARAA.