

Markscheme

May 2025

**Mathematics: applications and
interpretation**

Higher level

Paper 3

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Instructions to Examiners

Abbreviations

- M** Marks awarded for attempting to use a correct **Method**.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- R** Marks awarded for clear **Reasoning**.
- AG** Answer given in the question and so no marks are awarded.
- FT** Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg **M1**, **A2**.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, e.g. **M1A1**, this usually means **M1** for an **attempt** to use an appropriate method (e.g. substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies **A3**, **M2** etc., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the **AG** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award **FT** marks as appropriate but do not award the final **A1** in the first part.

Examples:

| | Correct answer seen | Further working seen | Any FT issues? | Action |
|----|---------------------|--|--|---|
| 1. | $8\sqrt{2}$ | 5.65685... (incorrect decimal value) | No. Last part in question. | Award A1 for the final mark (condone the incorrect further working) |
| 2. | $\frac{35}{72}$ | 0.468111... (incorrect decimal value) | Yes. Value is used in subsequent parts. | Award A0 for the final mark (and full FT is available in subsequent parts) |

3 Implied marks

Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then **FT** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is **(M1)A1**, it is possible to award full marks for *their* correct answer, **without working being seen**. For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (e.g. probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any **FT** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these **FT** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread and do not award the first mark, even if this is an **M** mark, but award all others as appropriate.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (e.g. probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- **MR** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for parts of questions are indicated by **EITHER . . . OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, **M** marks and intermediate **A** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come “*from the use of 3 sf values*”.

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an **A** mark to be awarded, arithmetic should be completed, and any

values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An

exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$.

However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^x$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate **A** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10 Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a)

$$(T =) \begin{pmatrix} 0.91 & 0.04 & 0.06 \\ 0.05 & 0.95 & 0.02 \\ 0.04 & 0.01 & 0.92 \end{pmatrix}$$

A1A1A1

Note: Award **A1** for each correct column.

[3 marks]

(b) recognizing the need to calculate T^5

(M1)

$$(T^5 =) \begin{pmatrix} 0.659 & 0.156 & 0.219 \\ 0.196 & 0.792 & 0.101 \\ 0.146 & 0.0514 & 0.680 \end{pmatrix} \left(\begin{pmatrix} 0.658687\dots & 0.156128\dots & 0.219028\dots \\ 0.195559\dots & 0.792469\dots & 0.101208\dots \\ 0.145753\dots & 0.0514021\dots & 0.679762\dots \end{pmatrix} \right) \text{ (A1)}$$

selects relevant cell (e.g. 0.219 seen separately)
21.9%

(M1)
A1

Note: Final answer must be a percentage. Follow through from their T , including the transpose.

[4 marks]

(c) attempt to find steady state vector (e.g. raising T to a high power)

(M1)

$$\begin{pmatrix} 0.342 \\ 0.432 \\ 0.225 \end{pmatrix} \left(\begin{pmatrix} 0.342342\dots \\ 0.432432\dots \\ 0.225225\dots \end{pmatrix} \right)$$

A1

[2 marks]

Note: Award **M1A0** if they express steady state vector as $\begin{pmatrix} 0.342 & 0.342 & 0.342 \\ 0.432 & 0.432 & 0.432 \\ 0.225 & 0.225 & 0.225 \end{pmatrix}$

(d) 34.2%

A1
[1 mark]

(e) require new steady state vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ such that $a \geq 0.40$ (seen anywhere) **(R1)**

any correct manipulation of Clover's entries in T (e.g. complementary increase and decrease of $T_{1,3}$ and $T_{3,3}$) **(A1)**

Note: This **A1** can be awarded independently of the **R1**.

$$(T =) \begin{pmatrix} 0.91 & 0.04 & 0.08 \\ 0.05 & 0.95 & 0.02 \\ 0.04 & 0.01 & 0.90 \end{pmatrix}$$

$$\begin{pmatrix} 0.403 \\ 0.459 \\ 0.138 \end{pmatrix}$$

when $C \rightarrow A$ is 13%

A1

Note: If candidates go on to state 13% - 6% = 7%, then ignore this and award **A1**. If their answer is just '7%' then award **A0**.

[3 marks]

(f) since $0.459 > 0.432$ **R1**
 therefore Bodega will benefit **A1**

Note: Do not award **ROA1**.

[2 marks]

(g) Accept any appropriate reason that shows engagement with the scenario.
 e.g.
 (Convergence to steady state vector takes a relatively long time and ...)

...during which time either of the three stores may notice a drop/influx of customers and change their advertising accordingly, thus changing T . **A1**

...during this time, a fourth (etc.) store could open, impacting the model. **A1**

...during this time, customers may move in and out of the region in a non-consistent rate. **A1**

Note: 'Customers may change their minds' or similar, award **A0**.

[1 mark]

(h) attempt to apply $u_n = u_1 + (n - 1)d$ **(M1)**

$$= 600 + 9 \times 30$$

$$= 870$$

A1
[2 marks]

(i) 720 seen anywhere A1
 $720 + 1.12 \times 30$ A1
 $= 753.6$ AG
[2 marks]

(j) attempts to sum geometric series with $r = 1.12$ (M1)
 $720 + 30(1.12 + 1.12^2 + \dots + 1.12^5)$ (A1)
 $= 933$ (933.455...) A1
[3 marks]

(k) **METHOD 1 (AS formula from booklet and adapted general formula from part (j))**

general formula for AS: $600 + 30(n - 1)$ (A1)

attempt to adapt GS formula (from part (j)) to include “ $n - 5$ ” (M1)

general formula for GS: $720 + 30 \left[\frac{1.12(1.12^{n-5} - 1)}{1.12 - 1} \right]$ (A1)

EITHER

attempts to solve GS – AS = 500 (M1)

$n = 17.5052\dots$ A1

Note: Other correct n values, depending on original expressions, are possible but only differing by an integer. **FT** within the question.

OR

creates list of both sequences (M1)

1080, 1530.87... **AND** 1110, 1661.77... A1

Note: If list method is used, award **A1** for the cross-over values clearly stated.

THEN

\Rightarrow week 18 A1

Note: Award at most **(A1)(M1)(A1)(M0)A0A1** for a correct answer with no working.

[6 marks]

METHOD 2 (general formula from part (j) and adapted AS formula)

general formula for GS: $720 + 30 \left[\frac{1.12(1.12^n - 1)}{1.12 - 1} \right]$ (A1)

attempt to express AS formula for weeks after fifth week (M1)

general formula for AS: $720 + 30n$ (A1)

Note: The n values can be offset (e.g $n-1$ or $n-5$ etc.) provided the approach is consistent between GS and AS.

EITHER

attempts to solve $GS - AS = 500$ (M1)

$n = 12.5052\dots$ A1

Note: Other correct n values, depending on original expressions, are possible but only differing by an integer. **FT** within the question.

OR

creates list of both sequences (M1)

1080, 1530.87... **AND** 1110, 1661.77... A1

Note: If list method is used, award **A1** for the cross-over values clearly stated.

THEN

\Rightarrow week 18 A1

Note: Award at most (A1)(M1)(A1)(M0)A0A1 for a correct answer with no working.

[6 marks]

[Total: 29 marks]

2. (a) $(P(X > 55) =) 0.208737\dots$ (A2)
 recognizing Binomial distribution (M1)
 $(P(Y \geq 5) =) 0.621 (0.621150\dots)$ A1

Note: Award (A2)(M1)A0 for an answer of 0.622 (from use of 0.209).

[4 marks]

- (b) (variance =) $\frac{3.7^2}{25}$ OR (SD =) $\frac{3.7}{5}$ (A1)
 $\bar{X} \sim N\left(52.0, \frac{3.7^2}{25}\right)$

$P(51 < \bar{X} < 53) = 0.823 (0.823417\dots)$

A2
 [3 marks]

- (c) $H_0 : \mu = 52.0$ A1
 $H_1 : \mu > 52.0$ A1

Note: Accept hypotheses in words if they are clearly expressed and ‘population mean’ is referred to. Do not accept $H_0 : \mu = \mu_0$ unless μ_0 is explicitly defined to be 52.0.

$p\text{-value} = 0.105055\dots$

$0.105 > 0.05$

therefore there is insufficient evidence to reject H_0

(i.e. the population mean weight of eggs is 52.0 g)

A2
 R1
 A1

Note: Follow through within the question for the R1A1, from their p -value seen. Award at most A0A0A2R1A0 if hypotheses are omitted or incorrect. Award the final A1 only if the null hypothesis is also correct and the conclusion is consistent with both the direction of the inequality and the alternative hypothesis.

[6 marks]

- (d) EITHER
 52.66 ± 1.5888 A1A1

Note: Award A1 for each correct value. Award at most A1A0 if the \pm is incorrect or omitted.

OR

$(51.1, 54.2) ((51.0712\dots, 54.2488\dots))$

A1A1

Note: Award A1 for each correct end point. Award at most A1A0 if interval notation is incorrect or omitted. Allow $51.1 \leq \mu \leq 54.2$

[2 marks]

- (e) (i) paired t -test **A1**
[1 mark]
- (ii) **EITHER** **A1**
 That the differences in weights are normally distributed.
OR **A1**
 The weights of eggs from any two chickens are independent. **[1 mark]**
- (f) $H_0 : \mu_d = 0$ **OR** $H_0 : \mu_2 - \mu_1 = 0$ **A1**
 $H_1 : \mu_d > 0$ **OR** $H_1 : \mu_2 - \mu_1 > 0$ **A1**
- p -value = 0.03377479... **A2**
 $0.0338 < 0.05$ **R1**
 (there is sufficient evidence that) the mean weight of the chickens' eggs has increased following the new feed **A1**

Note: The conclusion must be in context for the final **A1** to be awarded. Follow through within the question for the **R1A1**, from their p -value seen. Award at most **A0A0A2R1A0** if hypotheses are omitted or incorrect. Do not follow through from an incorrect part (e)(i).

[6 marks]

- (g) 2.39... (2.391555...) **A1**
[1 mark]
- (h) an attempt to find s_R^2 using the given expression and their part (g) **(M1)**
 $s_R^2 = 2.391555... \times 3.1789$
 $= 7.60$ **A1**
[2 marks]

[Total: 26 marks]