Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 3

21 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 15 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[55 marks]**.

2225 – 7308

Answer **both** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 29]

The following question explores how sequences, series and Markov chains may be used in modelling the number of customers in a commercial setting.

In a town, there are three stores: Aroma, Bodega and Clover.

Ashley is the manager of Aroma. She gathers data to determine whether there is significant movement of customers between the three stores over the course of one year.

She found that:

- 91% of Aroma customers stayed with Aroma, 5% moved to Bodega, and 4% moved to Clover.
- 95% of Bodega customers stayed with Bodega, 4% moved to Aroma, and 1% moved to Clover.
- 92% of Clover customers stayed with Clover, 6% moved to Aroma, and 2% moved to Bodega.

This information is used to form a transition matrix, $\boldsymbol{T}$.

(a) Write down the transition matrix $\boldsymbol{T}$. [3]

It is assumed that the movement of customers between stores remains constant from year to year.

(b) Determine the percentage of Clover customers expected to move to Aroma over a 5-year period. [4]

(c) Find the steady state vector for $\boldsymbol{T}$. [2]

(d) Hence, state the percentage of Clover customers expected to move to Aroma in the long term. [1]

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(Question 1 continued)

Ashley's initial findings suggested that 6% of Clover's customers moved to Aroma over the course of one year. Ashley is instructed to increase this figure so that at least 40% of Clover's customers move to Aroma **in the long term**.

It may be assumed that all other annual percentage changes remain the same, other than the percentage that stay with Clover.

(e) Determine the minimum integer percentage to which the 6% figure will need to be raised to achieve this objective, justifying your answer. [3]

(f) State, with an explanation, whether Bodega's manager would benefit from Ashley attracting more of Clover's customers. [2]

(g) Suggest a contextual reason why the annual percentage changes are unlikely to be constant from year to year. [1]

Ashley moves to a new store, Dusk, in a different town.

She notes that in her first week, Dusk had 600 customers and that every week, the number of customers increases by 30. Therefore, Ashley models the number of customers as an arithmetic sequence (**Model 1**).

(h) Use **Model 1** to find the number of customers Ashley can expect Dusk to have in her 10th week. [2]

Ashley wants to increase the number of customers at Dusk, so she introduces a loyalty scheme after her 5th week. She anticipates that the number of **new customers** she can expect each week should be 12% greater than the number of **new customers** the previous week (**Model 2**).

(i) Show that the number of customers in Ashley's 6th week, according to **Model 2**, is 753.6. [2]

(j) Use **Model 2** to find how many customers Ashley should expect in her 10th week. [3]

(k) By comparing the two models, determine the first week in which there would be an expected difference of at least 500 customers. [6]

2. [Maximum mark: 26]

The following question uses statistical tests to compare the weights of eggs in different situations.

Farmer Giles owns a chicken farm and sells eggs at the local market. He assumes that the weights of his eggs follow a normal distribution, with mean $52.0\,\text{g}$ and standard deviation $3.7\,\text{g}$.

Giles selects 25 eggs at random.

(a) Find the probability that at least 5 of these eggs weigh at least $55.0\,\text{g}$. [4]

(b) Calculate the probability that the mean weight of these 25 eggs lies within 1 gram of the population mean. [3]

In an effort to increase the mean weight of his eggs, Farmer Giles gives the chickens a new food, *Chick Crackle*, for one month. He then randomly selects 10 eggs and weighs them. His results, in grams, are shown in **Table 1**.

Table 1

54.5	52.4	50.4	54.3	54.2	53.1	50.2	51.8	52.2	53.5

Farmer Giles assumes the weights of all his eggs are still normally distributed but believes the mean and standard deviation of the weights may have changed.

(c) Perform a suitable test at the 5% significance level to investigate whether the mean weight of his eggs has increased from $52.0\,\text{g}$. [6]

(d) Find a 99% confidence interval for the mean weight of Giles's eggs, after feeding his chickens *Chick Crackle*. [2]

To analyse further the effect of *Chick Crackle*, Giles decides to choose 8 specific chickens. He weighs a randomly selected egg from each chicken before feeding them *Chick Crackle*. He then weighs another randomly selected egg from each chicken after feeding them *Chick Crackle* for one month, to determine whether the new feed has the required effect.

His results, in grams, are shown in **Table 2**.

Table 2

Chicken	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Egg weight before	53.8	51.5	49.3	52.3	46.4	55.0	51.2	52.4
Egg weight after	55.1	51.9	50.0	54.8	47.3	53.5	52.2	54.4

(e) (i) State a suitable test that Giles should use. [1]

(ii) State one assumption that Giles needs to make before carrying out the test. [1]

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(Question 2 continued)

(f) Perform the test at the 5% significance level, stating your conclusion in context. [6]

From the next village, Farmer Ray offers Farmer Giles a random sample of 10 eggs from his own chickens and requests that Giles carries out a t-test to see whose farm produces the heavier eggs.

Farmer Giles recalls from his statistics degree that a t-test is only valid if both population variances are the same.

(g) Given that Giles uses his data from **Table 1**, calculate the value of S_G^2, the unbiased estimator for the population variance of his eggs. [1]

He sets out to determine the maximum value of S_R^2, the unbiased estimator for the population variance of Ray's eggs, for which the t-test is considered valid. To do this, he uses a test called the F-test.

The test statistic for the F-test in this case is $F = \dfrac{s_R^2}{s_G^2}$.

The critical value at the 5% level for comparing two samples of 10 eggs is $F_{crit} = 3.1789$.

(h) Determine the maximum value of S_R^2 for the t-test to be valid. [2]