

Markscheme

May 2025

**Mathematics: applications and
interpretation**

Higher level

Paper 2

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Instructions to Examiners

Abbreviations

- M** Marks awarded for attempting to use a correct **Method**.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- R** Marks awarded for clear **Reasoning**.
- AG** Answer given in the question and so no marks are awarded.
- FT** Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg M1, A2*.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, *e.g. M1A1*, this usually means **M1** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies **A3, M2 etc.**, do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the **AG** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award **FT** marks as appropriate but do not award the final **A1** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	No. Last part in question.	Award A1 for the final mark (condone the incorrect further working)
2.	$\frac{35}{72}$	0.468111... (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award A0 for the final mark (and full FT is available in subsequent parts)

3 Implied marks

Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then **FT** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is **(M1)A1**, it is possible to award full marks for *their* correct answer, **without working being seen**. For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (e.g. probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any **FT** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these **FT** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread and do not award the first mark, even if this is an **M** mark, but award all others as appropriate.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (e.g. probability greater than 1, $\sin \theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscoopying of candidates’ own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a “show that” question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- **MR** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is ‘Hence’ and not ‘Hence or otherwise’ then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for parts of questions are indicated by **EITHER . . . OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, **M** marks and intermediate **A** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come “*from the use of 3 sf values*”.

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an **A** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$.

However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^x$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate **A** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) (i) 7 A1
(ii) 9 A1
[2 marks]
(b) (i) 2 A1
(ii) $1 + 2(n-1) (= 2n-1)$ (M1)A1

Note: (M1) for substituting their (b)(i) into the arithmetic sequence formula.

- (c) (i) 17 [3 marks] A1
(ii) $\frac{9}{2}(1+17)$ OR $\frac{9}{2}(2 \times 1 + (9-1) \times 2)$ OR $1 + 3 + \dots + 15 + 17$ (M1)
 $= 81$ A1

- (d) $\frac{n}{2}(1 + (2n-1))$ OR $\frac{n}{2}(2 \times 1 + (n-1) \times 2)$ (M1)
 $= n^2$ A1

- (e) 10 [2 marks] A1

- (f) $1 + 2 + 3 + \dots + n$ [1 mark]
 $= \frac{n}{2}(1 + n)$ (M1) A1

Note: Award (M1) for attempt for use of sum to n terms of an arithmetic sequence.

- (g) $\frac{n(1+n)}{2} + \frac{n(n-1)}{2}$ [2 marks] M1

Note: Award M1 for addition of answers to parts (f) and given expression.

$$= \frac{n}{2}[(1+n) + (n-1)] \quad \text{OR} \quad \frac{n + n^2 + n^2 - n}{2} \quad \text{M1}$$

Note: Award M1 for valid non-trivial attempt at further simplification of their sum.

$$= \frac{n}{2}(2n) \quad \text{OR} \quad \frac{2n^2}{2}$$

$$= n^2 \quad \text{A1}$$

Note: Award a maximum of M1M1A0 for ft from an incorrect part (f).

[3 marks]
Total [16 marks]

2. (a) 0.638 (0.637623...) and 4.10 (4.09811...) **A2A1**

Note: Award **A2** for one correct answer, accept 4.1. Award **A2A0** if coordinate pair is given.

[3 marks]

(b) (i) $\int_{0.638}^{4.1} 1.5^x dx$ OR $\int_{0.638}^{4.1} f(x) dx$

A1A1

Note: **A1** for function and dx , **A1** for limits. Award **A1** for $\int_{0.638}^{4.1} y dx$.

Condone the use of modulus within the integrand.

(ii) 9.81 (9.80788...)(from 3sf)

9.80 (9.79844...) (from full)

A2

Note: Accept 9.8 for **A2**.

(iii) correct expression for area

(A1)

$\int_{0.638}^{4.1} g(x) - f(x) dx$ OR $\int_{0.638}^{4.1} 6 - \frac{3}{x} - 1.5^x dx$

5.38 (5.38290...)

A1

Note: Accept 5.39

(c) $f'(x) = g'(x)$

[6 marks]

correct graphs of $f'(x)$ and $g'(x)$

(M1)

(M1)

Note: Award **M1** if $\frac{3}{x^2}$ seen.

$(k=)1.86$ (1.86406...)

A1

[3 marks]

Total [12 marks]

3. (a) $(A =) 2\pi r^2 + 2\pi r h$ A1A1

Note: Award **A1** for either term correct (even if unsimplified) and **A1** for all correct (including simplification and no extra term).

[2 marks]

(b) $V = \frac{2\pi r^3}{3} + \pi r^2 h$ A1A1

$V = \frac{2\pi r^3 + 3\pi r^2 h}{3}$ AG

Note: Award **A1** for one correct term, **A1** for both and completion to required form.

[2 marks]

- (c) 10000 substituted into (b)
correct rearrangement with at least one line of intermediate working leading to the given value.

M1

M1

$h = \frac{30000 - 2\pi r^3}{3\pi r^2}$ AG

[2 marks]

- (d) h substituted into their A

M1

$(A =) 2\pi r^2 + 2\pi r \left(\frac{30000 - 2\pi r^3}{3\pi r^2} \right)$

further consistent rearrangement that leads to **given** A

A1
[2 marks]

(e) $\frac{4\pi r}{3} - \frac{20000}{r^2}$ A1A1A1

Note: Award **A1** for $\frac{4\pi r}{3}$, **A1** for -20000 seen, and **A1** for a correct index for r in second term.

[3 marks]

(f) $\frac{dA}{dr} = 0$ (M1)

EITHER

rearrangement leading to $r^3 = \frac{15000}{\pi}$ (M1)

OR

graph of $\frac{dA}{dr}$ (M1)

THEN

$r = 16.8$ (16.8389...) OR $\sqrt[3]{\frac{15000}{\pi}}$ A1

Note: Accept correct graph of Area function leading to correct minimum value for r for **M1M1A1**.

$h = 0$ A1

[4 marks]

- (g) hemisphere A1

Note: Do not **FT** from incorrect answers to part (f).

[1 mark]

Total [16 marks]

4. (a) (since there are 5 vertices, 4 edges are required)
 start with A
 add E (AE = 0.86) **M1**
 add C (EC = 0.62)
 add B (CB = 0.74)
 add D (CD = 0.78) **A2**
- Note:** The **A2** is awarded for a correct Prim's algorithm. It cannot be split.
- A1**
- [4 marks]**
- (b) attempt to combine MST with AF **(M1)**
 LB = (length of minimum spanning tree) + (FA + FE)
 = 3 + 0.96 + 1.14
 = 5.1(0) (km) **A1**
- [2 marks]**
- (c) attempt to use the nearest neighbour algorithm **(M1)**
- Note:** Award **M1** for first 3 edges correct or 0.96, 0.86, 0.62 seen in order
- FA , AE , EC, CB, BD, DF OR 0.96, 0.86, 0.62, 0.74, 0.94, 2.24 **A1**
 UB = 6.36 (km) **A1**
- [3 marks]**
- (d) use A or C for LB and A, B or F for UB **A1A1**
 any attempt to convert distances to time **(M1)**
- LB = $(5.02 \div 28) + (5 \times 0.2) = 1.18 \text{ hrs } (1.17928\dots)$
 UB = $(6.36 \div 28) + (5 \times 0.2) = 1.23 \text{ hrs } (1.22714\dots)$
 $1.18 \leq T \leq 1.23$ **A1**
- [4 marks]**
- Total [13 marks]**

5. (a) 186 A1
[1 mark]
- (b) using midpoints within formula for mean M1

$$\frac{11 \times 103 + 14 \times 52 + 17 \times 31}{186}$$
12.8 (12.8387...) (mm) A1
[2 marks]
- (c) $\frac{42}{86}$ or $\frac{21}{43}$ seen (0.488372...) (A1)

$$\left(\frac{21}{43}\right)^2$$
method mark for squaring their probability (M1)

$$= 0.239 \text{ (0.238507...)} \left(= \frac{441}{1849} \right)$$
 A1
[3 marks]
- (d) choosing a chi-squared test for independence (M1)
 p -value = 0.127 (0.126687...) A1
 p -value > 0.05 therefore do not reject H_0 R1

Note: Accept p -value > significance level, as this is stated in the question.

there is no significant evidence that length category and sex are not independent A1

Note: Do not award **R0A1**. Accept length category and sex are independent. Allow internal follow through in this part for a consistent comparison and conclusion from their p -value.

[4 marks]

- (e) $H_0 : \phi = 0.45$; $H_1 : \phi > 0.45$ A1
choosing a binomial test (M1)

Note: can be implied by mention of Binomial, calculator notation, recognisable p -value

under the null hypothesis, if X is the number of males then $X \sim B(186, 0.45)$

p -value = $P(X \geq 100)$ (M1)

= 0.0101 (0.010103...) A1

since p -value < 0.05 we reject H_0

there is significant evidence that more than 45% of cane beetles are male. A1

Note: Do not award the final **A1** unless the correct p -value is seen.

[5 marks]

Total [15 marks]

6. (a) associated matrix is $\begin{pmatrix} 0 & 1 \\ -3 & -4 \end{pmatrix}$ (A1)

attempt to find eigenvalues (M1)

$$-\lambda(-4 - \lambda) + 3 = 0$$

eigenvalues are -3 and -1 (A1)(A1)

therefore the general solution for x is $Ae^{-3t} + Be^{-t}$ A1

[5 marks]

(b) (i) using initial condition for x , $A + B = 0$ A1

attempting to find derivative of general solution (M1)

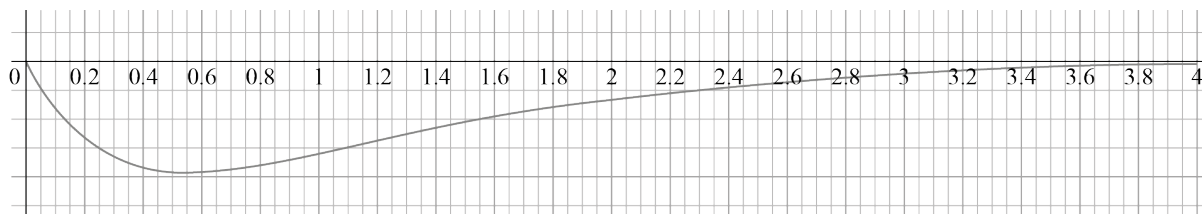
$$\frac{dx}{dt} = -3Ae^{-3t} - Be^{-t}$$

$$-3A - B = -1$$
 A1

$$A = \frac{1}{2}, B = -\frac{1}{2}$$

$$x = \frac{1}{2}e^{-3t} - \frac{1}{2}e^{-t}$$
 A1

(ii)



any curve starting at 0 and initially decreasing. A1

all else correct (minimum and some indication of POI / asymptote. A1

[6 marks]

(c) (i) recognition that minimum is relevant to the question (M1)

0.549 (minutes) (0.549306...) A1

(ii) any consideration of the y-coordinate of the minimum point (M1)

0.192 dollars (0.192450...) A1

[4 marks]

continued...

Question 6 continued

(d) attempting to write out the iterative relationship in terms of the variables (M1)

$$x_{n+1} = x_n + 0.1(y_n) \quad (A1)$$

$$y_{n+1} = y_n + 0.1(-4y_n - 3x_n + x_n \sin t_n) \quad (A1)$$

$$t_n = 0.1n$$

using $y_0 = x'_0 = -1$ (A1)

attempt to find x_{10} (M1)

Note: Award the **M1** for $x_1 = -0.1$ and $y_1 = -0.6$ seen as the first iterative stage.

$$x_{10} = -0.172 \text{ } (-0.171795...) \quad A1$$

Note: a common error may be 0.104 (0.104330...).

[6 marks]

Total [21 marks]

7. (a) (i) 0.4 A1
(ii) 0.6 A1
[2 marks]
- (b) (i) $0.4^4 = 0.0256$ (M1)A1
(ii) considering $M^4 = \begin{pmatrix} 0.5422 & 0.5341 \\ 0.4578 & 0.4659 \end{pmatrix}$ (M1)(A1)
required probability is therefore 0.542 A1
[5 marks]
- (c) attempting Mv with given eigenvector M1

$$\begin{pmatrix} 0.4 & p \\ 0.6 & 1-p \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

correct matrix multiplication

$$\begin{pmatrix} 0.4 - p \\ -0.4 + p \end{pmatrix}$$
 A1
writing as a multiple of given vector

$$= (0.4 - p) \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$
 (A1)
therefore the eigenvalue is $0.4 - p$ A1
[4 marks]
- (d) consider eigenvector with eigenvalue 1 $Mv = v$ (M1)

$$\begin{pmatrix} 0.4 & p \\ 0.6 & 1-p \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

turn into equations:
 $0.4x + py = x$ OR $0.6x + (1-p)y = y$ A1
also use $x + y = 1$ A1
then $x + \frac{0.6x}{p} = 1$

$$x = \frac{p}{p + 0.6}$$
 A1
[4 marks]
- (e) graphical, inequality or equality approach to solve $\frac{p}{p + 0.6} \geq 0.6$ (M1)
 $p = 0.9$ A1
[2 marks]
Total [17 marks]