

# Markscheme

**May 2025**

**Mathematics: applications and  
interpretation**

**Higher level**

**Paper 2**

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## Instructions to Examiners

### Abbreviations

- M** Marks awarded for attempting to use a correct **Method**.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- R** Marks awarded for clear **Reasoning**.
- AG** Answer given in the question and so no marks are awarded.
- FT** Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

### Using the markscheme

#### 1 General

Award marks using the annotations as noted in the markscheme *eg M1, A2*.

#### 2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award **M0** followed by **A1**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, *e.g. M1A1*, this usually means **M1** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and **A1** for using the **correct** values.
- Where there are two or more **A** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award **A0A1A1**.
- Where the markscheme specifies **A3, M2** etc., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the **AG** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award **FT** marks as appropriate but do not award the final **A1** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... (incorrect decimal value)	No. Last part in question.	Award <b>A1</b> for the final mark (condone the incorrect further working)
2.	$\frac{35}{72}$	0.468111... (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award <b>A0</b> for the final mark (and full <b>FT</b> is available in subsequent parts)

### 3 Implied marks

Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

### 4 Follow through marks (only applied after an error is made)

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then **FT** marks should be awarded for *their* correct answer, even when working is not present.

**For example:** following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is **(M1)A1**, it is possible to award full marks for *their* correct answer, **without working being seen**. For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further **A** marks can be awarded for work which uses the error, but **M** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (e.g. probability greater than 1,  $\sin \theta = 1.5$ , non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any **FT** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these **FT** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

## 5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular misread. Use the **MR** stamp to indicate that this has been a misread and do not award the first mark, even if this is an **M** mark, but award all others as appropriate.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (e.g. probability greater than 1,  $\sin \theta = 1.5$ , non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- **MR** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

## 6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for parts of questions are indicated by **EITHER . . . OR**.

## 7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, **M** marks and intermediate **A** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

## 8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come “*from the use of 3 sf values*”.

**Simplification of final answers:** Candidates are advised to give final answers using good mathematical form. In general, for an **A** mark to be awarded, arithmetic should be completed, and any

values that lead to integers should be simplified; for example,  $\sqrt{\frac{25}{4}}$  should be written as  $\frac{5}{2}$ . An

exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example,  $\frac{10}{4}$  may be left in this form or written as  $\frac{5}{2}$ .

However,  $\frac{10}{5}$  should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g.  $4e^{2x} \times e^{3x}$  should be simplified to  $4e^{5x}$ , and  $4e^{2x} \times e^{3x} - e^{4x} \times e^x$  should be simplified to  $3e^{5x}$ . Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so  $x(x+1)$  and  $x^2 + x$  are both acceptable.

**Please note:** intermediate **A** marks do NOT need to be simplified.

## 9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

## 10 Presentation of candidate work

**Crossed out work:** If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

**More than one solution:** Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) ( $k =$ ) 5 **A1**  
**[1 mark]**

(b) (i) attempt to substitute into the mean formula with a sum of at least two correct products **and** equating to 2.2 (or  $2.2(p+27)$  if denominator not seen) or equating to “mean”. Equation may be seen in part b(ii). **(M1)**

$$2.2 = \frac{0 \times 3 + 1 \times 11 + 2 \times 7 + 3 \times 5 + 4 \times p + 5 \times 1}{3 + 11 + 7 + 5 + p + 1} \quad \text{OR} \quad 2.2 = \frac{45 + 4p}{p + 27} \quad \text{A1}$$

(ii) ( $p =$ ) 8 **A1**

**Note:** Follow through from part (b)(i) only if their value of  $p$  is a whole number.

**[3 marks]**

(c) finding the median ( $= 2$ ), quartiles ( $Q_1 = 1$  and  $Q_3 = 4$ ), or interquartile range of the data  $IQR = 3$  from the 2024 tournament **(A1)**

**Note:** Award **(A1)** for any correctly identified/labelled value referring to 2024 data as listed in the question. Accept “Range from 0 to 5” without reference to 2024.

Any two of the following statements: **R1R1**  
 The median number of goals per game **in both years** is 2.  
**In both years**, (the lower quartile is 1 and the upper quartile is 4) the IQR is 3.  
**Both** distributions are (roughly) skewed to the right

**Note:** Award **R1** for each correct statement. Each statement must reference values from both the box and whisker diagram **and** the data from the 2024 tournament to award **R1**. (e.g. “the median is 2” earns **R0**, “the median for both years is 2” earns **R1**.) If an incorrect statement is made, award at most **R1R0**.

**[3 marks]**

(d)  $B$  and  $C$  **A1A1**

**Note:** Award **A1** for each correct event, if two events are selected.  
 Award **A1A0** for two correct events and one additional incorrect event selected.  
 Award **A0A1** for an answer of “ $A$ ,  $D$ , and  $E$ ” from misreading the meaning of  $F'$ .  
 Award **A0A0** for any other combination of events selected.

**[2 marks]**

(e)  $\frac{5}{17} \left( \frac{10}{34} \right)$

**A1A1**

**Note:** Award **A1** for correct numerator, **A1** for correct denominator. Award at most **A1A0** for a decimal answer (0.294117...) if no fraction is shown. Do not award **A1ft** for their denominator if their answer to part (a) or part (b)(ii) is not a whole number.

**[2 marks]**

(f)  $\frac{1}{35} \times \frac{3}{34}$

**(A1)(A1)(M1)**

**Note:** Award **A1** for each correct probability, **M1** for multiplying their two probabilities. Do not award any marks if their answer to part (a) or part (b)(ii) is not a whole number.

0.00252 (0.00252100...,  $\frac{3}{1190}$ , 0.252%)

**A1**

**[4 marks]**

**Total [15 marks]**

2. (a) (i) attempt to substitute any given coordinates into midpoint formula **(M1)**

$$(12, 12.5) \text{ or } \left(12, \frac{25}{2}\right) \quad \text{A1}$$

**Note:** Award **A0** if parentheses are omitted.

- (ii) attempt to find gradient of BC using coordinates of B and C **(M1)**

$$(\text{gradient of BC} =) \frac{12-13}{16-8} \left( = -\frac{1}{8} = -0.125 \right) \quad \text{A1}$$

$$(\text{gradient of } \perp = ) 8 \quad \text{A1}$$

**Note:** Follow through within question part for final **A1** from their gradient of BC only if the gradient is seen.

- (iii) **EITHER**

$$8(x-12) = y-12.5 \quad \left( 8(x-12) = y-12\frac{1}{2} \right) \quad \text{A1}$$

**OR**

$$y = 8x - 83.5 \quad \left( y = 8x - 83\frac{1}{2} \right) \quad \text{A1}$$

**[6 marks]**

- (b) (i) **EITHER**

B is expected to respond since (14, 10) lies in the same cell (of the Voronoi diagram) as fire station B **R1**

**OR**

B is expected to respond since (14, 10) is closer to B than to any other fire station **R1**

**OR**

B is expected to respond since it's the closest (nearest responder) to (14, 10) **R1**

**OR**

Fire station B will respond as the fire is within B's boundaries/area/region. **R1**

**Note:** Do not award **R1** unless the statement refers to both the distance (closer) to B (or located in the B cell) **and** the responder (point B or fire station B) For example, award **R0** for "Point B is closer to (14, 10)" as there is no reference to a responder.

- (ii) Award **R1** for any viable reason such as:

- fire station B might have already responded to a different fire
- a different fire station might have more manpower or equipment available to fight the fire
- a different fire station might respond to assist fire station B if the fire is large
- the Voronoi diagram only considers straight-line distances, so a different fire station might have a shorter driving route to the fire

**R1**  
**[2 marks]**

(c) (i) attempt to calculate the length DB (M1)

a correct expression for DB A1

$$(DB =) \sqrt{\left(11\frac{1}{3} - 16\right)^2 + \left(7\frac{1}{6} - 12\right)^2}$$

DB = 6.71855 AG

(ii) **METHOD 1**

$$(DC =) \sqrt{\left(11\frac{1}{3} - 8\right)^2 + \left(7\frac{1}{6} - 13\right)^2}$$
A1

DC = 6.71855 therefore, the fire is equidistant from all 3 fire stations R1

so, any of the fire stations would be expected to respond AG

**Note:** Do not award **A0R1**. Do not award **R1** if **AG** statement is not seen.

**METHOD 2**

The intersection of bisector AB and bisector BC :

$y = 8x - 83.5$  and  $y = 0.5x + 1.5$  have an intersection at  $D\left(11\frac{1}{3}, 7\frac{1}{6}\right)$  A1

**Note:** Calculator solve is acceptable but D must be clearly identified from a calculation of intersecting lines.

Since D is the intersection of the bisector lines, the fire at D is equidistant from all 3 fire stations R1

so, any of the fire stations would be expected to respond AG

**Note:** Do not award **A0R1**. Do not award **R1** if **AG** statement is not seen.

**[4 marks]**

- (d) (i) **METHOD 1: Dividing into two triangles**
- $(0.5 \times 20 + 1.5 =) 11.5$  seen in an area calculation (A1)
- $(20 - 7.75 =) 12.25$  **OR**  $(20 - 11.3333... =) 8.66666...$  seen (A1)
- substitution into triangle area formula for either triangle (M1)
- $(A =) 0.5 \times (20 - 7.75) \times 7.16666... + 0.5 \times 11.5 \times (20 - 11.3333...)$  (A1)
- $93.7291... \left( \frac{4499}{98} \right)$  (sq. units)
- 94 (sq. units) A1

**Note:** Answer must be given as a whole number to award final A1.  
Award (A1)(A1)(M1)(A1)A0 for an answer of 93.7 without working.

- METHOD 2: Dividing vertically into triangle and trapezoid**
- $(0.5 \times 20 + 1.5 =) 11.5$  seen in an area calculation (A1)
- attempt to substitute into area of trapezoid formula (M1)
- $(A =) 0.5 \times (7.16666... + 11.5) \times (20 - 11.3333...)$

**Note:** Lengths of parallel sides must be seen to award the M1.

- correct calculation of right triangle area (A1)
- $(A =) 0.5 \times (11.3333... - 7.75) \times 7.16666...$
- $(A =) 0.5 \times (11.3333... - 7.75) \times 7.16666... + 0.5 \times (7.16666... + 11.5) \times (20 - 11.3333...)$  (A1)
- $93.7291... \left( \frac{4499}{98} \right)$  (sq. units)
- 94 (sq. units) A1

**Note:** Answer must be given as a whole number to award final A1.  
Award (A1)(M1)(A1)(A1)A0 for an answer of 93.7 without working.

**METHOD 3: Dividing horizontally into triangle and trapezoid**

$(0.5 \times 20 + 1.5 =) 11.5$  seen in an area calculation **(A1)**

attempt to substitute into area of trapezoid formula **(M1)**

$$(A =) 0.5 \times [(20 - 7.75) + (20 - 11.3333\dots)] \times 7.16666\dots$$

**Note:** Lengths of parallel sides must be seen to award the **M1**.

correct calculation of right triangle area **(A1)**

$$(A =) 0.5 \times (20 - 11.3333\dots) \times (11.5 - 7.16666\dots)$$

$(A =) 0.5 \times (20 - 11.3333\dots) \times (11.5 - 7.16666\dots) + 0.5 \times [(20 - 7.75) + (20 - 11.3333\dots)] \times 7.16666\dots$  **(A1)**

$$93.7291\dots \left( \frac{4499}{98} \right) \text{ (sq. units)}$$

94 (sq. units) **A1**

**Note:** Answer must be given as a whole number to award final **A1**.  
Award **(A1)(M1)(A1)(A1)A0** for an answer of 93.7 without working.

**METHOD 4: Dividing into two triangles and rectangle**

$(0.5 \times 20 + 1.5 =) 11.5$  seen in an area calculation **(A1)**

$(20 - 11.3333\dots)$  **OR**  $(11.3333\dots - 7.75)$  **OR**  $(11.5 - 7.16666\dots)$  seen **(A1)**

substitution into triangle area formula. **(M1)**

$0.5 \times (11.3333\dots - 7.75) \times 7.16666\dots + (20 - 11.3333\dots) \times 7.16666\dots + 0.5 \times (11.5 - 7.16666\dots) \times (20 - 11.3333\dots)$  **(A1)**

$$93.7291\dots \left( \frac{4499}{98} \right) \text{ (sq. units)}$$

94 (sq. units) **A1**

**Note:** Answer must be given as a whole number to award final **A1**.  
Award **(A1)(A1)(M1)(A1)A0** for an answer of 93.7 without working.

**METHOD 5: Calculus**

A and C mid point (13, 10.5)

$$(\text{gradient of AC} = ) \frac{13-8}{8-18} = -\frac{5}{10}$$

(gradient of  $\perp$ ) = 2

$$\text{AC bisector } 2(x-13) = y-10.5 \quad \text{OR} \quad y = 2x - 15.5$$

**A1**

attempt to integrate their equation and given equation

**(M1)**

$$\text{area} = \int_{7.75}^{11.3} 2x - 15.5 \, dx + \int_{11.3}^{20} 0.5x + 1.5 \, dx$$

**(A1)(A1)**

**Note:** Award **A1** for each correct expression with correct limits.

$$93.7291... \left( \frac{4499}{98} \right) (\text{sq. units})$$

94 (sq. units)

**A1**

**Note:** Answer must be given as a whole number to award final **A1**.  
Award **A1(M1)(A1)(A1)A0** for an answer of 93.7 without working.

(ii) (1 sq unit = 2.5 km  $\times$  2.5 km =) 6.25 (km<sup>2</sup>)

**(A1)**

**Note:** Award **A1** for 6.25 seen.

$$(94 \times 6.25 \text{ km}^2)$$

$$588 (587.5) (\text{km}^2)$$

**A1**

**Note:** Accept: 586 (585.807...) (km<sup>2</sup>) from (93.7291...  $\times$  6.25 km<sup>2</sup>).

**[7 marks]**  
**[Total: 19 marks]**

3. (a) evidence of using the values 0,20,40,60,... for their  $x$  data **(M1)**

$$f(x) = -0.0989x^2 + 45.9x - 67.8 \quad \mathbf{A2}$$

$$(f(x) = -0.0989285\dots x^2 + 45.925x - 67.7857\dots)$$

**Note:** Accept models using full population,  $f(x) = -98.9285\dots x^2 + 45925x - 67785.7\dots$

Award **(M0)A2** for  $f(x) = -0.0989285\dots x^2 + 421.853\dots x - 444457$  or  $f(x) = -98.9285\dots x^2 + 421853.5\dots x - 444457428.6\dots$  from finding regression without converting years. Award at most **(M1)A1A0** for a correct model with one rounding error or misread. Award at most **(M1)A0A0** for models with multiple rounding errors. Award at most **(M1)A1A0** if the answer is not given in equation form.

**[3 marks]**

- (b) **EITHER**  
 $R^2$  is (very) close to 1 **OR** 0.98843 is (very) close to 1 **R1**

**OR**  
 $R^2$  shows that the model explains 98.843% of the total variation **R1**

**THEN**  
 therefore, it supports Ethan's proposal **A1**

**Note:** Do not award **ROA1**. Accept "Yes" for the A mark as long as reasoning is correct. To award the R1, a comparison to 1 must be seen if not using the percentage method.

**[2 marks]**

- (c) **EITHER**  
 the negative value of  $c$  is unrealistic as you cannot have a negative population **R1**

**OR**  
 the negative value of  $a$  is unrealistic because it suggests that the population of Los Angeles will decrease back to 0 at some point in the future / be negative in the future / was negative before 1900 **R1**  
**[1 mark]**

- (d) (i)  $(f'(110) =) 24.2 (24.1607\dots)$  **A1**

**Note:** Accept 24.1(24.142) from use of 3sf values in model. Accept 400(400.089...) or 400,000 (400,089.28 ...) from use of a model from part (a) where years were not converted.

(recognition that value predicts the) annual growth (in 2010 as)  
 24,200(24,160.7...) per year **OR** 24.2(24.1607...) thousands per year **A1**

**Note:** Allow FT within question part for the final A mark for a correct interpretation of their incorrect value for  $f'(110)$ . Do not follow through for interpreting a value for  $f(110)$ .

(ii) **EITHER**

population growth is clearly slowing down / only averaged a growth of 10 700 per year between 2000 and 2020

**A1**

validity questioned since the model's predicted growth per year in 2010 is significantly higher

**R1**

**OR**

total population change between 2000 and 2020 is 214,000 and the predicted change using  $f'(110)$  over that time frame 483,200

**A1**

validity supported since prediction is in the correct order of magnitude / low percentage error for the predicted population of 2020 .

**R1**

**Note:** For either of the arguments, the response must include a comparison of their answer for part (d)(i) with specific values from the actual data. FT marks can only be awarded in this question part if value if  $15 < f'(110) < 30$  . .

Do not award FT marks in this question part for responses that include a model where the years were not converted as their value in (d)(i) would tell them the population rate of change in the year 110.

**[4 marks]**

(e) the model predicts the slowing growth rate of the population over the long term / the population is trending towards a carrying capacity

**R1**  
**[1 mark]**

(f) (i) attempt to differentiate  $g(x)$  using chain/quotient rule

**(M1)**

$$(g'(x) =) \frac{2800e^{-0.05x}}{(1+14e^{-0.05x})^2}$$

**A1A1**

(ii) recognition to find maximum of  $g'(x)$

**(M1)**

(maximum occurs when  $x =$ ) 52.7806...

**A1**

(therefore, greatest growth rate occurred during the year) 1952

**A1**

**Note:** FT within the question part for the final A1 as long as the candidate's given value for  $x$  is not a whole number (must be evidence of rounding down to match the context of the problem).

**[6 marks]**  
**[Total 17 marks]**

4. (a)  $\sqrt{(-0.40)^2 + (-0.35)^2 + (0.1)^2}$  (A1)  
 0.541  $\left(0.540832\dots, \frac{3\sqrt{13}}{20}\right)$  (kilometres per minute) A1

[2 marks]

- (b) any two of the following: A1A1
- flying in a straight line
  - flying in an overall NW direction
  - flying towards the west at  $1.4 \text{ km min}^{-1}$  (units required)
  - flying towards the north at  $1.65 \text{ km min}^{-1}$  (units required)
  - flying at a constant altitude / at an altitude of 7 kilometres / parallel to the ground
  - passes through a point that is 5 kilometres east, 2 kilometres south, and 7 kilometres above the ground

**Note:** Statement must be about the path and not the speed/velocity. Do not accept any statement referring to moving towards the east at  $-1.4 \text{ km min}^{-1}$ .

[2 marks]

(c) (i)  $6 + 0.1t_1 = 7(+0t_1)$  (A1)  
 $t_1 = 10$  (min) (accept a final answer of "10:10 (am)") A1

(ii) substitution of  $t_1 = 10$  into either vector equation (M1)

airplane (will have a position vector of)  $\begin{pmatrix} 5 \\ -2 \\ 7 \end{pmatrix} + (10) \begin{pmatrix} -1.4 \\ 1.65 \\ 0 \end{pmatrix} = \begin{pmatrix} -9 \\ 14.5 \\ 7 \end{pmatrix}$  A1

birds (will have a position vector of)  $\begin{pmatrix} -11 \\ 30 \\ 6 \end{pmatrix} + (10) \begin{pmatrix} -0.40 \\ -0.35 \\ 0.05 \end{pmatrix} = \begin{pmatrix} -15 \\ 26.5 \\ 7 \end{pmatrix}$  A1

**Note:** All three of the above marks can be awarded to candidates who work with only the  $x$ -coordinates or only the  $y$ -coordinates. The complete position vectors do not need to be found.

$x$ - or  $y$ - coordinates are different at the (only) time their  $z$ -coordinate is equal R1

**Note:** The response must include a reference to the  $x$ - or  $y$ - coordinates being different to award the R1. Simply referring to the two position vectors is not sufficient. A statement saying that the 'north' and/or 'east' positions are different is sufficient. Allow FT within question part for final R mark for a correction statement related to their position vectors

therefore airplane and birds will not collide AG  
 [6 marks]

(d) 
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 7 \end{pmatrix} + (t_2 + 20) \begin{pmatrix} -1.4 \\ 1.65 \\ 0 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 7 \end{pmatrix} + (20) \begin{pmatrix} -1.4 \\ 1.65 \\ 0 \end{pmatrix} \quad (\mathbf{A1})$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -23 \\ 31 \\ 7 \end{pmatrix} + t_2 \begin{pmatrix} -1.4 \\ 1.65 \\ 0 \end{pmatrix} \quad \mathbf{A1}$$

[2 marks]

(e) attempt to find difference between any coordinate at time  $t_2$  (M1)

$x = -95 + 1.3t_2 - (-23 - 1.4t_2)$  and

$y = 32 + 1.45t_2 - (31 + 1.65t_2)$  and

$z = 1.5 + 0.2t_2 - 7$  (A1)

attempt to substitute their expressions into distance formula (M1)

$$d = \sqrt{(2.7t_2 - 72)^2 + (-0.2t_2 + 1)^2 + (0.2t_2 - 5.5)^2}$$

(using GDC, minimum distance =) 4.33(4.32565...) (kilometres) A1

therefore the jet and the airplane will break the rule (since  $4.33 < 5$ ) R1

**Note:** Do not award **R1** unless value for minimum distance is seen. If no adjustment is made to the original passenger airplane vector equation, award at most **(M1)(A0)(M1)(A0)R1**.

Allow FT within question part for final R mark as long as  $0 < d < 10$  and their value of  $d$  comes from an attempt to use the distance formula.

**[5 marks]**  
**[Total 17 marks]**

5. (a) correct substitution into distance formula (A1)  
 $\sqrt{(13-13)^2 + (25-1)^2 + (7-0)^2}$  OR  $\sqrt{0^2 + 24^2 + 7^2}$

**Note:** Accept omission of  $(13-13)^2$  or  $0^2$ .

(AC =) 25 (cm)

**A1**  
**[2 marks]**

- (b) **METHOD 1**

AB || y-axis AND BC || z-axis (AB = 24*k* AND BC = 7*j*)

**A1**

AB perpendicular to BC since y-axis is perpendicular to z-axis OR  $AB \cdot BC = 0$

**R1**

therefore  $\Delta ABC$  must be a right-angled triangle.

**AG**

**METHOD 2**

24 and 7 seen

**A1**

$$24^2 + 7^2 = 25^2 \text{ OR } 625 = 625 \text{ OR (angle B =) } \cos^{-1}\left(\frac{7^2 + 24^2 - 25^2}{2(7)(24)}\right) = 90^\circ$$

**R1**

**Note:** Accept  $AB^2 + BC^2 = 625$  as long as the length of AB and of BC are defined.

therefore  $\Delta ABC$  must be a right-angled triangle

**AG**  
**[2 marks]**

- (c) width of 9 or correct cross-section area (84) seen

**(A1)**

correct substitution into volume formula

**(A1)**

$$V = \frac{1}{2}(24)(7)(9)$$

(Volume =) 756 cm<sup>3</sup>

**A1**

**Note:** Do not award final **A1** if correct units are not included.

**[3 marks]**

(d) (i) correct use any trig ratio (M1)

$$\tan \theta = \frac{7}{24} \quad \text{OR} \quad \sin \theta = \frac{7}{25} \quad \text{OR} \quad \cos \theta = \frac{24}{25}$$

( $\theta =$ ) 16.3° (16.2602...°) OR 0.284(0.283794...) (radians) A1

(ii) **METHOD 1**  
recognition that the width of the door stop is still 9 (M1)

$$625 = \frac{1}{2}bh(9)$$

$$bh = \frac{1250}{9}$$

$$\tan(16.2602\dots^\circ) = \frac{h}{b} \quad \text{OR} \quad \tan(0.283794\dots) = \frac{h}{b} \quad (M1)$$

$$b^2 = \frac{1250}{9 \tan(16.2602\dots^\circ)} \quad \text{OR} \quad b^2 = \frac{1250}{9 \tan(0.283794\dots)} \quad (A1)$$

( $b = AX =$ ) 21.8 (21.8217...) (cm) A1

**METHOD 2 (area similarity)**  
recognition that the new side face will be similar to ABC (M1)

$$\text{proportionality of area } k^2 = \frac{625}{756} \quad (A1)$$

$$\text{proportionality of length } k = \sqrt{\frac{625}{756}} = 0.909241$$

new base length  $AX = 24k$  (M1)

21.8 (21.8217...) (cm) A1

[6 marks]

(e) multiplying 625 by 1.1 or 1.2 or 0.025 (M1)

$$625 \times 1.1 \times 0.025 \times 1.2 \quad (A1)$$

(price =) 20.63 (20.625) (USD) A1

[3 marks]

[Total 16 marks]

6. (a)  $\frac{dh}{dt} = k \times \frac{h}{v}$  (A1)

attempt to substitute given values into their equation with a constant of proportionality M1

(When  $t = 0$ )  $-5 = k \times \frac{1350}{27}$  OR  $-5 = k \times \frac{1350}{27000}$

$k = -\frac{1}{10}$  A1

$\frac{dh}{dt} = -\frac{h}{10v}$  AG

[3 marks]

(b)  $\frac{dh}{dt} = -\frac{h}{10(27+0.3t)}$  (A1)

**Note:** Follow through within question part for all remaining marks for responses that lose the initial A mark here from omitting the 10  $\left(\frac{dh}{dt} = -\frac{h}{27+0.3t}\right)$

attempt to separate variables to setup integration (M1)

$\int \frac{1}{h} dh = \int -\frac{1}{10(27+0.3t)} dt$  (A1)

**EITHER**

$\ln h = -\frac{1}{3} \ln(27+0.3t) + c$  A1

**Note:** This A1 can be awarded even if  $+c$  is not seen.

$h = A(27+0.3t)^{-\frac{1}{3}}$  (A1)

$A = 4050$  (from using  $t = 0$  and  $h = 1350$ ) (A1)

$h = 4050(27+0.3t)^{-\frac{1}{3}}$  A1

**Note:** The follow through answer here for those that omit 10 in the first step is.

$h = 79716150(27+0.3t)^{-\frac{10}{3}}$

**OR**

$$\ln h = -\frac{1}{3} \ln(270 + 3t) + c \quad \text{A1}$$

**Note:** This **A1** can be awarded even if  $+c$  is not seen.

$$h = A(270 + 3t)^{\frac{1}{3}} \quad \text{(A1)}$$

$$A = 4050\sqrt[3]{10} \quad (8725.46\dots) \quad (\text{from using } t = 0 \text{ and } h = 1350) \quad \text{(A1)}$$

$$h = 8730(270 + 3t)^{\frac{1}{3}}$$

$$\left( h = 8725.46\dots(270 + 3t)^{\frac{1}{3}}, 4050\sqrt[3]{10}(270 + 3t)^{\frac{1}{3}} \right) \quad \text{A1}$$

**[7 marks]**

(c) attempt to setup equation or inequality comparing the number of virus particles to the number of healthy cells **(M1)**

recognition to multiply expression for  $v$  by 1000 **OR** dividing expression for  $h$  by 1000 **(M1)**

$$1000(27 + 0.28t) \geq 100(4380(27 + 0.28t)^{-0.36}) \quad \text{(A1)}$$

**Note:** Accept an equation.

$$t = 216 \quad (216.253\dots) \quad (\text{hours}) \quad \text{A1}$$

**Note:** Accept 217 as a contextual answer. Award at most **(M1)(M0)A0A1FT** for an answer of 50100(50136.06\dots) or 50137 from correct solution of inequality where the candidate has not appreciated the 1000.

**[4 marks]**

**[Total 14 marks]**

7. (a) **METHOD 1**

attempt to represent area under curve with integral **(M1)**

$$\int_{0.1}^1 \frac{2}{3} \cos^{-1}\left(\frac{1}{10x}\right) dx \quad \text{(A1)}$$

0.780871... seen **(A1)**

1 – 0.780871...

0.219 (0.219128...) (m<sup>2</sup>) **A1**

**METHOD 2**

recognition to split area into three areas **(M1)**

$(1 \times (1 - 0.980419...)) = 0.0195807... \text{ seen AND}$

$(0.1 \times 0.980419... =) 0.0980419... \text{ seen}$  **(A1)**

$$\int_{0.1}^1 \left( 0.980419... - \frac{2}{3} \cos^{-1}\left(\frac{1}{10x}\right) \right) dx \quad \text{(M1)}$$

0.0195807... + 0.0980419... + 0.101505...

0.219 (0.219128...) (m<sup>2</sup>) **A1**

**METHOD 3**

attempt to rearrange to make  $x$  a function of  $y$  **(M1)**

$$x = \frac{1}{10 \cos\left(\frac{3y}{2}\right)}$$

Integral upper bound  $\frac{2}{3} \cos^{-1}(0.1) = 0.980419 \text{ seen}$  **(A1)**

attempt to represent area to left of curve with sum of integral **AND** rectangle area **(M1)**

$$\int_0^{0.980419...} \frac{1}{10 \cos\left(\frac{3y}{2}\right)} dy + 1 \times (1 - 0.980419...)$$

**Note:** Must see attempt at both an integral and a rectangle area to award **M1A0**.

0.219 (0.219128...) (m<sup>2</sup>) **A1**

**[4 marks]**

(b) recognition of rectangles forming cylinders when rotated

(M1)

**Note:** Evidence of recognition includes:

- the word 'cylinder' written in work
- $\pi r^2 h$  seen anywhere
- $\pi(1)^2$  or  $\pi(0.3)^2$  multiplied to anything

attempt to set up integral for middle section

(M1)

**Note:** Attempt includes any integral that includes  $\pi$  and squaring some form of the given expression, regardless of whether they have attempted to rearrange to solve for  $x$ . It is also not necessary to see the  $dy$  or proper limits here for the M1.

attempt to rearrange expression to solve for  $x$  (possibly seen in part(a))

(M1)

**Note:** If this is seen in part (a), it must be seen within an integral here to award the mark.

value of 0.980419... or 0.0195807... seen anywhere (possibly seen in part (a))

A1

correct volume expression seen for one of the cylinders

A1

fully correct expression

A1

$$(V =) \pi(1)^2(0.0196) + \int_0^{0.980} \pi \left( \frac{1}{10 \cos\left(\frac{3y}{2}\right)} \right)^2 dy + \pi(0.3)^2(0.075)$$

$$\left( \pi(1)^2(0.0195807\dots) + \int_0^{0.980419\dots} \pi \left( \frac{1}{10 \cos\left(\frac{3y}{2}\right)} \right)^2 dy + \pi(0.3)^2(0.075) \right)$$

[6 marks]

(c) 0.291 (0.291110...) (m<sup>3</sup>)

A2

**Note:** FT from their expression in part (b) as long as their expression contains three parts and their final value is positive.

[2 marks]

[Total 12 marks]