Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

2 hours

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2225 – 7307

15 pages

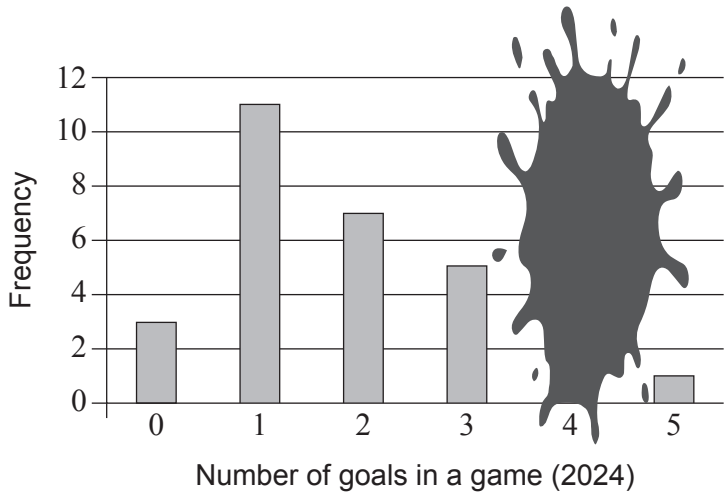
Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

Paul has a bar graph for the total number of goals scored in each game of a soccer tournament in 2024. The bar graph is shown below, however the frequency of 4 goals in a game is unreadable.

Paul uses this bar graph to create a frequency table.

Bar graph



Frequency table

Number of goals in a game (2024)	Frequency
0	3
1	11
2	7
3	k
4	p
5	1

(a) Write down the value of k. [1]

Paul knows that the mean number of goals per game scored during the tournament was 2.2.

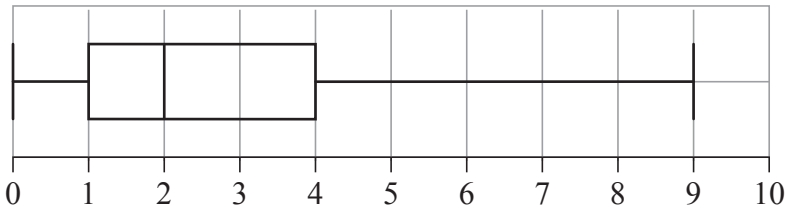
(b) (i) Write down an equation for the mean in terms of p.

(ii) Determine the value of p. [3]

(This question continues on the following page)

(Question 1 continued)

Data for the number of goals per game in the **2025 soccer tournament** are shown in the following box and whisker diagram.



Number of goals in a game (2025)

After comparing the box and whisker diagram from the 2025 tournament with the frequency table from the 2024 tournament, Paul concludes that the distribution of goals is consistent between the two tournaments.

(c) State two observations that support Paul's conclusion using values from the data to compare any **two** of:

range, symmetry, median, and interquartile range. [3]

Paul plans to watch all the games from the **2024** tournament in a random order.
He will watch each game once.

For the first game he watches, he defines event F as:

"scoring either 0 goals or exactly 1 goal".

(d) Write down the event(s) from the table that are equivalent to F'. There may be more than one correct event. [2]

Event	Description
A	Scoring exactly 2 goals in this game
B	Scoring more than 1 goal in this game
C	Scoring at least 2 goals in this game
D	Scoring either 0 goals or exactly 1 goal in all games except this game
E	Not scoring 0 goals or exactly 1 goal in any game

(e) If exactly 1 goal was scored in the first game Paul watches, write down the probability that exactly 1 goal was scored in the second game he watches. Give your answer as a fraction. [2]

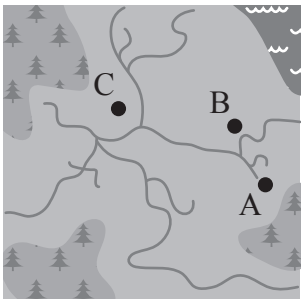
(f) Calculate the probability that 5 goals were scored in the first game that Paul watches **and** 0 goals were scored in the second game he watches. [4]

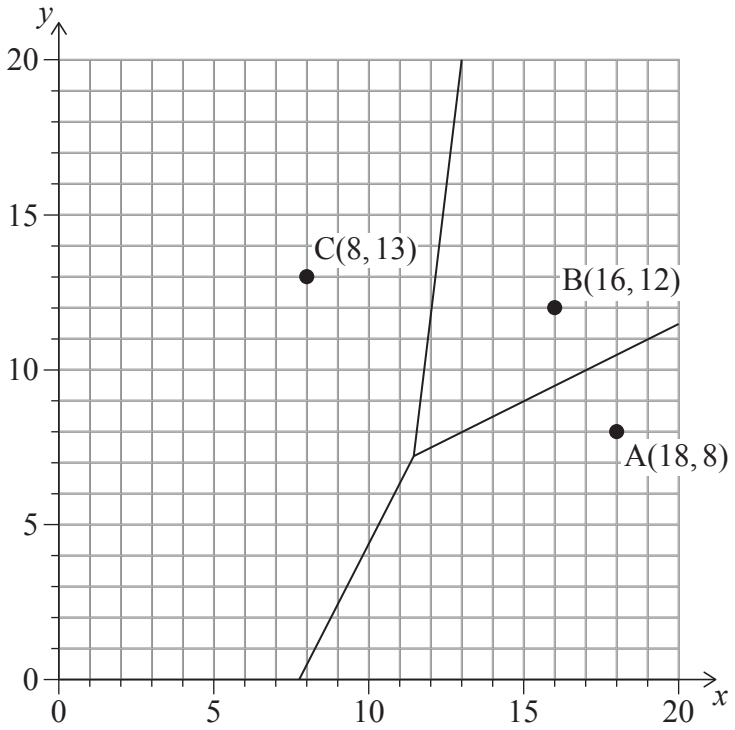
2. [Maximum mark: 19]

The locations of three fire stations within a 50 by 50 kilometre rural area of Japan are shown on the following map.

A Voronoi diagram can be used to determine the part of the rural area for which each fire station is responsible. The Voronoi diagram for the three fire stations is shown on the coordinate grid beside the map, where 1 unit represents $2.5\,\text{km}$.

Map





(a) (i) Find the midpoint of $[\text{BC}]$.

(ii) Find the gradient of the perpendicular bisector of $[\text{BC}]$.

(iii) Hence find the equation of the edge between sites B and C. [6]

(b) (i) Identify the fire station that is expected to respond, based on the Voronoi diagram, if a fire is reported at a location with coordinates $(14, 10)$. Justify your response.

(ii) Suggest a reason why a different fire station might respond to this fire. [2]

(This question continues on the following page)

(Question 2 continued)

A fire is reported at a location D with coordinates $\left(11\frac{1}{3}, 7\frac{1}{6}\right)$. The distance of D from A on the Voronoi diagram, to six significant figures, is 6.71855 units ($16.7964\,\mathrm{km}$ on the map).

(c) (i) Show that the distance of D from B, to six significant figures, is also 6.71855 units.

(ii) Show that any of the three fire stations would be expected to respond to the fire reported at D. [4]

The equation of the edge between fire stations A and B is $y = 0.5x + 1.5$. The edge between fire stations A and C has an x-intercept of 7.75.

(d) (i) Determine the area, to the nearest square unit, of the region within the given Voronoi diagram for which fire station A is responsible.

(ii) Hence, calculate the actual area, in square kilometres, of this region. [7]

3. [Maximum mark: 17]

Ethan and Avery are researching population data for the city of Los Angeles to create a model predicting population values. They collect the following data.

Population data for the city of Los Angeles

Year	Population (thousands)
1900	102
1920	577
1940	1504
1960	2479
1980	2967
2000	3685
2020	3899

Ethan proposes the population can be modelled using quadratic regression to find a function of the form $f(x) = ax^2 + bx + c$, where x is the number of years after 1900.

(a) Find the equation of Ethan's model. [3]

Ethan finds the coefficient of determination for his model is 0.98843 to five significant figures.

(b) State whether the coefficient of determination supports Ethan's proposal. Justify your answer. [2]

(c) Comment on the validity of Ethan's model with reference to one of the parameters in the equation. [1]

(d) (i) Find the value of $f'(110)$ and interpret this value in context.

(ii) By considering the population changes in the table, use the value found in part (d)(i) to comment on the validity of Ethan's model. [4]

(This question continues on the following page)

(Question 3 continued)

Avery proposes that the population instead follows a logistic model of the form

$$g(x) = \frac{4000}{1 + 14\ \mathrm{e}^{-0.05x}}$$

where x is the number of years after 1900.

(e) State a reason why it may be valid to use Avery's proposal to predict the future population. [1]

(f) (i) Find $g'(x)$.

(ii) Hence find the year, according to Avery's model, during which the greatest population growth rate occurred. [6]

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4. [Maximum mark: 17]

Lizjerika works in an air traffic control tower. At 10:00 am, she detects a passenger airplane and a flock of birds whose flight paths are given by the following equations:

$$\text{Airplane: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 7 \end{pmatrix} + t_1 \begin{pmatrix} -1.4 \\ 1.65 \\ 0 \end{pmatrix} \qquad \text{Birds: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -11 \\ 30 \\ 6 \end{pmatrix} + t_1 \begin{pmatrix} -0.40 \\ -0.35 \\ 0.1 \end{pmatrix}$$

where the x-coordinate represents displacement east of the control tower, the y-coordinate represents displacement north of the control tower, and the z-coordinate represents height (above sea level).

Distances are measured in kilometres and t_1 is the time, in minutes, since 10:00am.

(a) Determine the speed, in kilometres per minute, at which the flock of birds is travelling. [2]

(b) By making two statements, describe the path of the airplane in context. [2]

(c) (i) Find the time at which the birds and the airplane will have the same height.

(ii) Using your answer to part (c)(i), explain why the birds will not collide with the airplane. [6]

At 10:20 am, Lizjerika detects a jet whose movement is given by the following equation,

$$\text{Jet: } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -95 \\ 32 \\ 1.5 \end{pmatrix} + t_2 \begin{pmatrix} 1.3 \\ 1.45 \\ 0.2 \end{pmatrix}$$

where t_2 is the time, in minutes, since 10:20am.

(d) Find the vector equation of the motion of **the passenger airplane** in terms of t_2. [2]

An air traffic law requires that the jet and the airplane must be more than 5 kilometres from each other at all times.

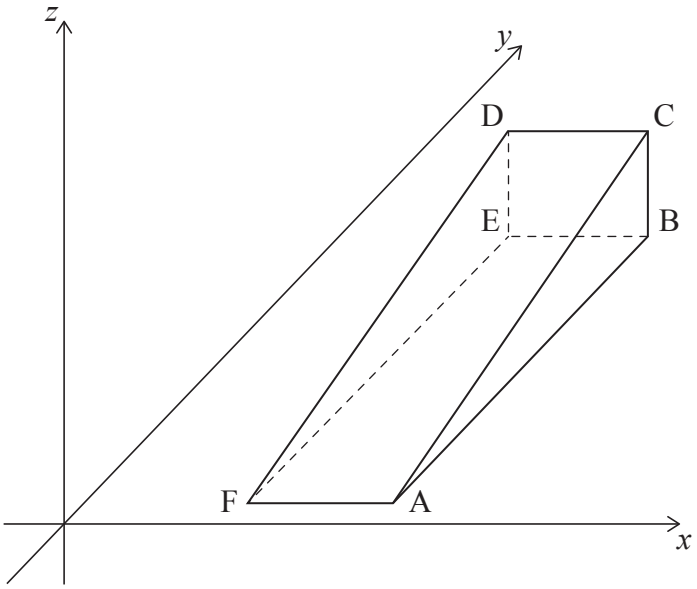
(e) Determine whether the jet and the airplane will break this law if they follow their current paths. [5]

5. [Maximum mark: 16]

AirFlow Industries is designing a doorstop in the shape of a triangular prism, $ABCDEF$.

The points $A(13, 1, 0)$, $B(13, 25, 0)$ and $C(13, 25, 7)$ are the coordinates of the vertices of one of the triangular faces of the prism. Point $D(4, 25, 7)$ is another vertex as shown in the following diagram. All the measurements are in centimetres.

diagram not to scale



(a) Find AC. [2]

(b) Show that triangle ABC is a right-angled triangle. [2]

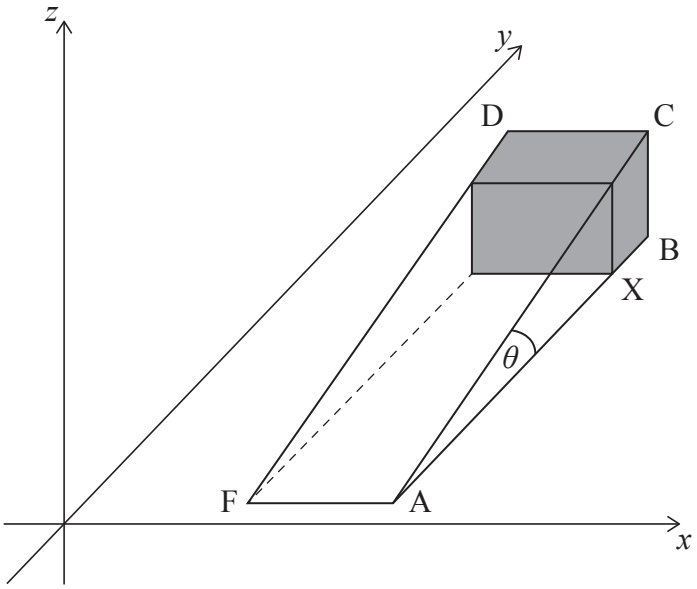
(c) Find the volume of material needed to make the doorstop. [3]

(This question continues on the following page)

(Question 5 continued)

To lower the cost of the doorstop, AirFlow decides to reduce its volume to $625\,\text{cm}^3$ by removing the shaded section shown in the following diagram. The remaining triangular faces are still right-angled triangles.

diagram not to scale



(d) (i) Find the value of θ.

(ii) Hence, or otherwise, find AX. [6]

The material needed to make the doorstop costs 0.025 US dollars (USD) per cubic centimetre. The production of each doorstop requires 10% more material than its final volume, due to wastage in the production process.

To make a profit, the company will sell the doorstop for 20% more than the cost of the material.

(e) Determine the price, to two decimal places, the company will charge for the **new** doorstop. [3]

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6. [Maximum mark: 14]

Rea is investigating how the number of healthy skin cells changes following infection by a virus. At the beginning of the investigation, she inserts $27\,000$ virus particles into a sample of 1350 healthy skin cells.

Let h be the number of healthy skin cells at time t, where t is the number of hours after the investigation has begun. Let v be the number of virus particles, **in thousands**, at time t.

Rea finds that the rate of change, $\frac{\mathrm{d}h}{\mathrm{d}t}$, is proportional to $\frac{h}{v}$.

(a) Given that $\frac{\mathrm{d}h}{\mathrm{d}t} = -5$ at time $t = 0$, show that $\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{h}{10v}$. [3]

Rea models the number of virus particles, in thousands, after t hours as $v = 27 + 0.3t$.

(b) Find an expression for the number of healthy skin cells at time t. [7]

Rea asks her colleague Artem to attempt the same investigation. Artem's models differ from those of Rea.

Artem models the number of virus particles, in thousands, after t hours as $v = 27 + 0.28t$ and finds that $h = 4380\,(27 + 0.28t)^{-0.36}$.

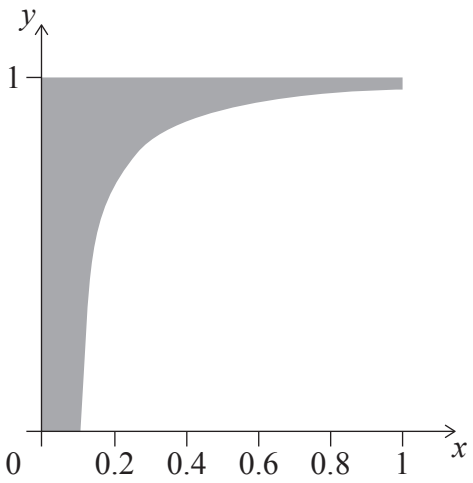
(c) Using **Artem's** models, predict how many hours it will take for the number of virus particles to be at least 100 times the number of healthy skin cells. [4]

7. [Maximum mark: 12]

Bobby is designing a table that will be 1 metre high. The table will have a flat circular top with a radius of 1 metre. To begin the design, he uses the curve of

$$y = \frac{2}{3}\cos^{-1}\left(\frac{1}{10x}\right), \text{ for } 0.1 \leq x \leq 1,$$

to shape the underside of the table, where x and y are measured in metres, as shown.



(a) Find the area of the shaded region. [4]

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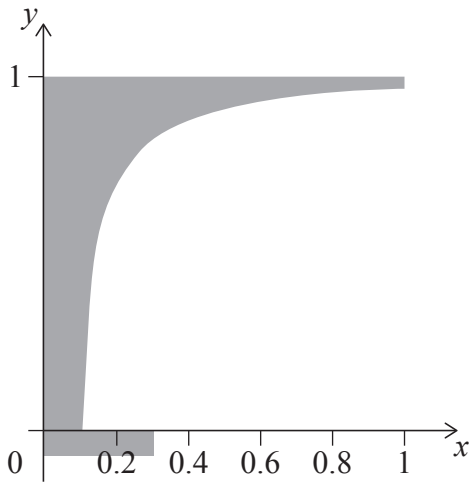
(Question 7 continued)

To create the base of the table, Bobby adds a rectangle with a height of 0.075 metres and a width of 0.3 metres to the bottom of the figure, as shown.



To finish the design of the table, he rotates the entire shaded region through 2π about the y-axis.

(b) Find the three expressions necessary to determine the total volume of this table. [6]

(c) Hence find the total volume of the table. [2]

References:

3. Los Angeles Almanac, n.d. *Historical General Population City & County of Los Angeles, 1850 to 2020* [online] Available at: https://www.laalmanac.com/population/po02.php [Accessed 4 June 2024]. Source adapted.