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Mathematics: applications and interpretation
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

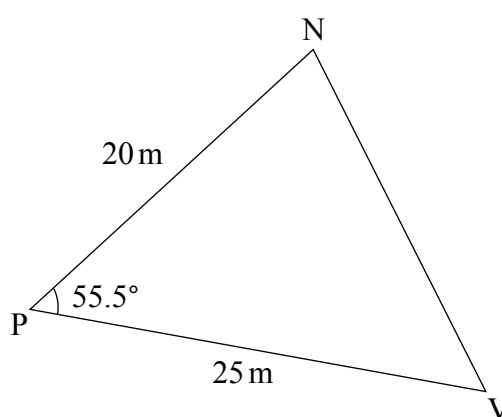


Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 9]

Three points N, P, and V are shown on the following diagram. NP is 20 metres, PV is 25 metres and $\angle VP\hat{N}$ is 55.5° .

diagram not to scale



- (a) Find NV. [3]
- (b) Find $P\hat{N}V$. [3]
- (c) Hence or otherwise, find the shortest distance between P and [NV]. [3]

(This question continues on the following page)





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will not be marked.



2. [Maximum mark: 5]

The loudness of a sound, L , measured in decibels (dB) is determined by the intensity of the sound, I , measured in watts per square metre (Wm^{-2}). The relationship between loudness and intensity can be expressed using the logarithmic function

$$L = 10 \log_{10} \left(\frac{I}{I_0} \right), \quad I > 0$$

where I_0 is the reference intensity (the intensity of the least audible sound to the human ear).

The reference intensity I_0 is 10^{-12}Wm^{-2} .

The intensity of sound on a busy street is 10^{-5}Wm^{-2} .

(a) Calculate the loudness of the sound. [2]

The sound of a jet engine reaches a loudness of 185 dB.

(b) Determine the intensity of its sound. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$, $k \in \mathbb{Z}$. [3]

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3. [Maximum mark: 7]

Prakash is the leader of a customer service team and is interested in determining whether there is a relationship between a customer's satisfaction level and the type of service interaction they have experienced.

He collects data from a random sample of 250 customers and tracks their satisfaction level after three types of service interactions: in-person, online chat bots and website contact forms.

He categorizes the satisfaction levels as satisfied, neutral and dissatisfied.

He records the data in the following table.

		Satisfaction level		
		Satisfied	Neutral	Dissatisfied
Type of service interaction	In-person	35	30	23
	Online chat bots	31	39	23
	Website contact forms	19	28	22

Prakash performs a χ^2 test for independence at the 5% significance level.

The critical value is 9.488.

The null hypothesis, H_0 , is the satisfaction level and the type of service interaction are independent.

- (a) State the alternative hypothesis for this test. [1]
- (b) Find the degrees of freedom for this test. [1]
- (c) Find χ^2_{calc} , the chi-squared test statistic. [2]

Prakash concludes that there is sufficient evidence to reject the null hypothesis.

- (d) (i) State whether Prakash is correct. Justify your answer.
- (ii) Write down the conclusion for this test in context. [3]

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(Question 3 continued)

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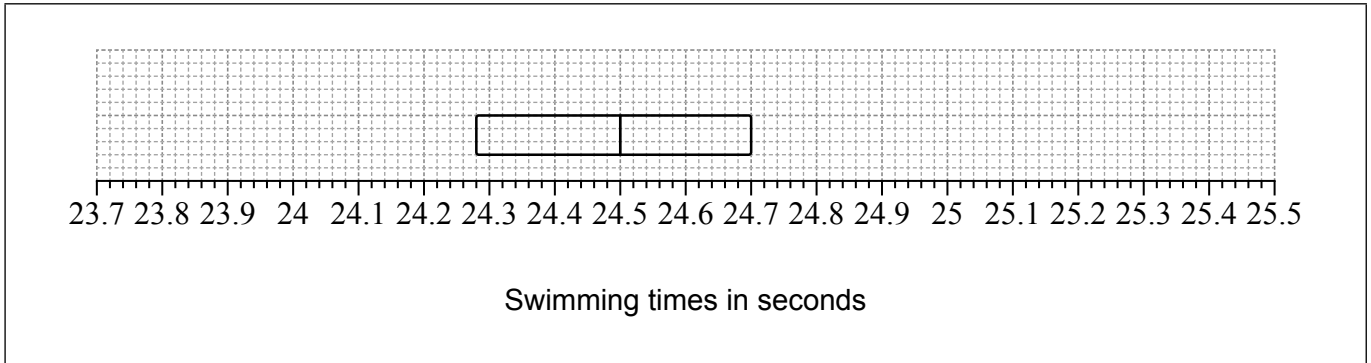
32EP07

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4. [Maximum mark: 7]

The times, in seconds, for the fastest 16 women in a 50 m freestyle swimming championship event were recorded. All swimmers recorded different times.

Part of a box and whisker diagram for these times is shown in the following diagram.



- (a) Write down the number of swimmers who took more than 24.70 seconds to complete the race. [1]
- (b) Find the interquartile range (IQR) for the data. [2]

An outlier is defined as a value that satisfies one of the following:

- more than $1.5 \times \text{IQR}$ below the lower quartile
- more than $1.5 \times \text{IQR}$ above the upper quartile.

Of the 16 women, the two fastest swimmers took 23.96 and 24.12 seconds and the two slowest women took 25.12 and 25.40 seconds to complete the race.

- (c) (i) Show that only one of these times is an outlier.
- (ii) Complete the box and whisker diagram above. [4]

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(Question 4 continued)

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5. [Maximum mark: 8]

Sofia takes out a loan for 50 000 euros. She pays a nominal interest rate of 4.2% per year, compounded monthly. She must pay back the loan in 25 years through regular monthly payments, made at the end of each month.

- (a) Find the amount Sofia must pay back each month. Give your answer correct to two decimal places.

[2]

At the end of eight years, Sofia wins the lottery and pays back the remainder of the loan early in one single payment.

- (b) (i) Find how much Sofia pays back in this single payment. Give your answer correct to two decimal places.
- (ii) Find the amount of money saved by Sofia by paying back the loan early.

[6]

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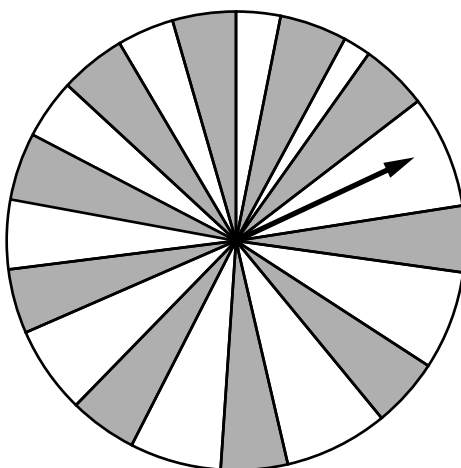
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6. [Maximum mark: 6]

A circular disk has 10 white sectors and 10 shaded sectors. The size of the angles, made at the centre of the circle by the white sectors, form an arithmetic sequence. The common difference of this sequence is 3° and the first term is 6° .

An arrow is attached to the centre of the circle as shown in the following diagram.



The arrow can be spun and has an equal chance of stopping at any point.

- (a) Find the probability that the arrow will stop in a white sector. [3]

In a game, the arrow is spun 4 times and the player wins if the arrow stops in a white sector at least 3 times.

- (b) Find the probability that the player wins the game. [3]

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(Question 6 continued)

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32EP13

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7. [Maximum mark: 7]

A team of researchers is using a model to predict the relative happiness of different countries. To do this, a value x is calculated based on easily measured parameters, for example, life expectancy, or available social support. It is assumed that higher values of x indicate greater happiness.

To test the model a survey is conducted in six countries, A, B, C, D, E and F. In these countries the level of happiness is assessed directly using questionnaires and given a score y , out of 10, with higher scores indicating greater happiness.

To select the countries for the survey, all countries are divided into three equal groups based on wealth and two countries are chosen randomly from each group.

(a) Write down the name of this type of sampling.

[1]

The results of the survey, along with the value obtained from the model, are given in the following table.

Country	A	B	C	D	E	F
Value from the model (x)	12.3	15.2	14.1	18.5	20.1	19.2
Happiness score (y)	5.2	7.3	6.2	6.9	8.0	7.2

The researchers will accept the model is a valid predictor of happiness score if the Pearson's product-moment correlation coefficient, r , is greater than 0.8.

(b) (i) Find the value of r .

(ii) Hence state whether the model can be regarded as a valid predictor of happiness score.

[3]

(c) Find the equation of the regression line y on x .

[1]

For a particular country $x = 17.2$.

(d) Use the regression line to predict the happiness score for this country.

[2]

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(Question 7 continued)

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32EP15

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8. [Maximum mark: 5]

The matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ transforms a point A to a point B.

The matrix $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$ transforms the point B to a point C with coordinates (4, 7).

Find the coordinates of A.

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9. [Maximum mark: 6]

Consider the curve $y = 4x\sqrt{3x-2}$ for $x \geq \frac{2}{3}$.

(a) Find $\frac{dy}{dx}$. [3]

(b) Hence show that the equation of the tangent to the curve at the point (6, 96) is $y = 25x - 54$. [3]

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10. [Maximum mark: 8]

A satellite orbiting the Earth experiences a gravitational force, $F(\text{N})$, which varies inversely as the square of its distance, d (kilometres), from the centre of the Earth.

When $d = 7000$, $F = 40\,000$.

- (a) (i) Find an equation for F in terms of d .
- (ii) Find the value of F when d is 3500 km. [4]
- (b) (i) Identify which **two** of the following graphs would form a straight line.
- A. F against d
 - B. F against d^2
 - C. F against d^{-2}
 - D. $\log_{10} F$ against $\log_{10} d$
 - E. $\log_{10} F$ against d
 - F. F against $\log_{10} d$
- (ii) For each of these two graphs, find the value of the gradient of the line formed. [4]

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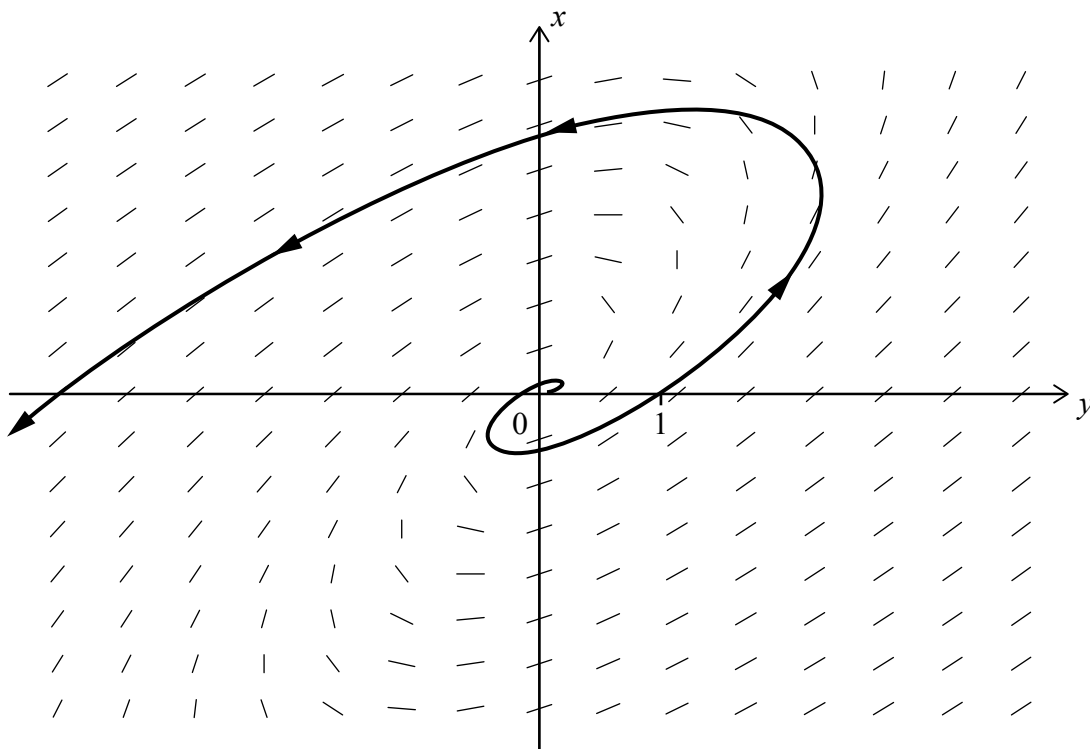


11. [Maximum mark: 6]

Consider the system given by the coupled differential equations

$$\begin{aligned}\frac{dx}{dt} &= 3x - 2y \\ \frac{dy}{dt} &= 4x - y\end{aligned}$$

The following phase portrait shows the trajectory that passes through the point $(1, 0)$ at time $t = 5$.



(a) Write down which one of the following could be an eigenvalue for the system. [1]

- A. 1
- B. $2i$
- C. $1 + 2i$
- D. $-1 + 2i$

(b) Use Euler's method with a step length of 0.1 to find the approximate values of x and y when $t = 5.5$. [5]

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(Question 11 continued)

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32EP21

Turn over

12. [Maximum mark: 5]

Tajinder believes that the number of cars arriving at an intersection every minute follows a Poisson distribution. Under this assumption he decides to test the null hypothesis that the mean is 12.4 cars per minute against the alternative hypothesis that the mean is greater than 12.4.

The test consists of observing the intersection for 10 minutes and then rejecting the null hypothesis if more than 130 cars arrive.

- (a) (i) State the meaning of a Type I error. [4]
- (ii) Find the probability of a Type I error for Tajinder's test. [4]
- (b) Write down one reason why a Poisson distribution might not be a suitable model for the number of cars arriving at the intersection. [1]

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13. [Maximum mark: 9]

The growth of a fungus on a dish can be modelled by the differential equation

$$\frac{dP}{dt} = 0.2(1 - P), \quad 0 \leq P < 1$$

where P is the proportion of the dish that is covered with fungus and t is the time in days from when the fungus began to form.

(a) Assuming that $P = 0$ when $t = 0$, show that $P = 1 - e^{-0.2t}$. [7]

(b) Find the time at which the fungus covers 90% of the dish. [2]

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14. [Maximum mark: 8]

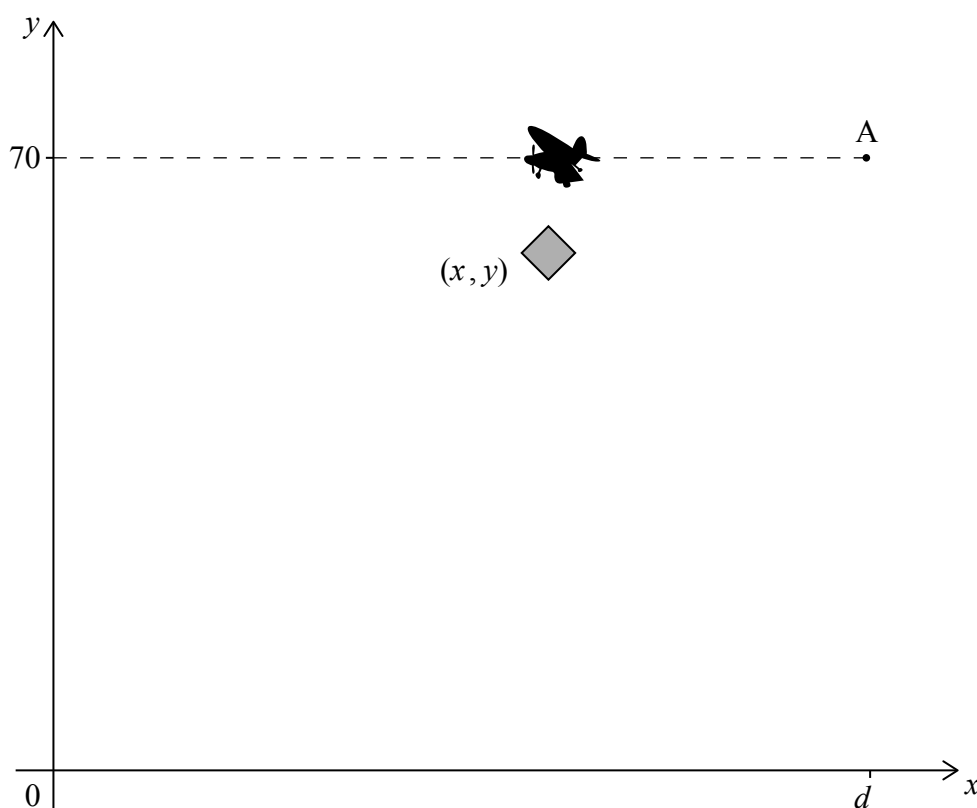
A plane needs to drop a relief package for a remote village. The villagers have marked out a point O in the middle of a field where the package should land.

The package is released from the plane at point A when it is a horizontal distance of d metres from O .

The position vector of the package t seconds after release is given by the vector $\begin{pmatrix} x \\ y \end{pmatrix}$,

where x is the horizontal displacement of the package from O and y is the vertical height of the package above the ground. This is shown in the following diagram.

diagram not to scale



(This question continues on the following page)



The acceleration of the package is $\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} \text{ms}^{-2}$ and its velocity at the point of release is $\begin{pmatrix} -20 \\ 0 \end{pmatrix} \text{ms}^{-1}$.

- (b) Find the value of d , given that the package lands at O. [6]

[illegible]

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15. [Maximum mark: 6]

A Markov chain has two states, A and B. The transition matrix for the Markov chain is

$$\begin{array}{cc} & \begin{array}{c} A \qquad B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \left(\begin{array}{cc} p & 1-q \\ 1-p & q \end{array} \right). \end{array}$$

One of the eigenvalues for this matrix is equal to 1 .

- Find the eigenvector corresponding to the eigenvalue of 1. [3]
- Hence, find an expression, in terms of p and q , for the proportion of time spent in state A. [2]
- Hence or otherwise, find the proportion of time spent in state A when $p = 0.3$ and $q = 0.6$. [1]

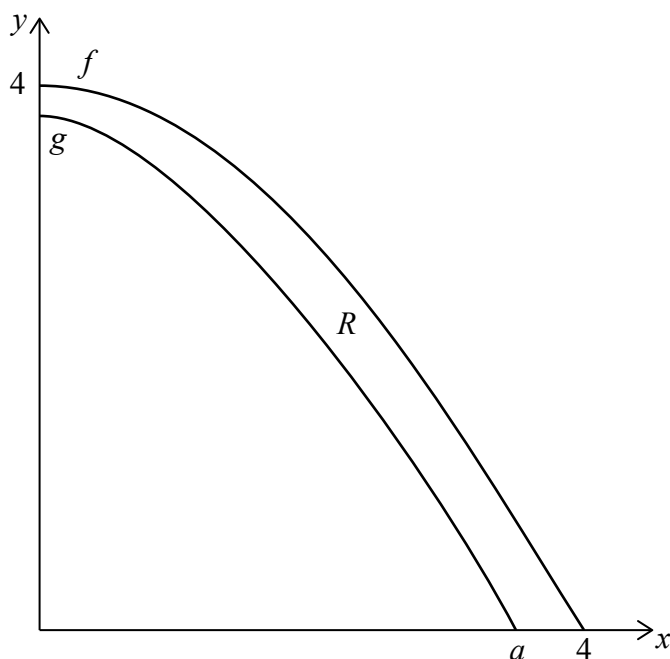
[illegible]

16. [Maximum mark: 8]

The graph of f is defined by $f(x) = 4 \cos\left(\frac{\pi}{8}x\right)$, $0 \leq x \leq 4$.

The graph of $y = g(x)$ is formed by translating the graph of f 0.3 units to the left and 0.2 units down. The graph of g intersects the x -axis at $(a, 0)$ and its domain is then restricted to $0 \leq x \leq a$.

These curves are shown on the following graph where units are in centimetres.



(a) Write down an expression for $g(x)$. [2]

(b) Find the value of a . [1]

A wall-mounted light shade can be modelled by the solid formed when the region between the two curves, R , is rotated through π radians about the x -axis.

(c) (i) Write down an expression for the volume of the solid formed.

(ii) Hence find the volume of material used in the light shade. [5]

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