2225 – 7311M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2025

Mathematics: applications and interpretation

Higher level

Paper 1

25 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) after 10 years

N: 120
I%: 3.6
PV: 0
PMT: (-)1500
P/Y: 12
C/Y: 12 ***(M1)(A1)***

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, award ***A1*** for all entries correct except PV.

FV: 216 278.58 (euros) ***A1***

Note: Award ***A0*** for a negative answer or answer not given to 2 decimal places.

[3 marks]

(b) **METHOD 1 (C.I. formula)**

invests: 116 278.58 ***(M1)***

after 15 years: $116278.58\times(1+\frac{4.5}{400})^{60}$ ***(M1)(A1)***

FV: 227 515.92 (euros) ***A1***

Note: Award the first ***M1*** for subtracting *their* FV from part (a) even if the value of the investment is negative.
Award the second ***M1*** for use of the C.I. formula.
Award first ***A1*** for all entries correct except PV.
Allow full follow through from part (a).
Award final ***A0*** for FV not given to 2 decimal places unless this has already been penalized in (a).
Accept 227 515.93 from use of unrounded value.

METHOD 2 (financial app)

invests: 116 278.58 ***(M1)***

THEN

EITHER		**OR**		**OR**	
N:	60	*N:*	15	*N:*	180
I%:	4.5	*I%:*	4.5	*I%:*	4.5
PV:	116 278.58	*PV:*	116 278.58	*PV:*	116 278.58
PMT:	0	*PMT:*	0	*PMT:*	0
P/Y:	4	*P/Y:*	1	*P/Y:*	12
C/Y:	4	*C/Y:*	4	*C/Y:*	4

(M1)(A1)

continued…

Question 1 continued

Note: Award the first ***M1*** for subtracting *their* FV from part (a) even if the value of the investment is negative.
Award ***(M1)*** for an attempt to use a financial app in their technology with a least two entries seen, award ***A1*** for all entries correct except PV.

THEN

FV: 227 515.92 (euros) ***A1***

Note: Award ***A0*** for a negative answer or answer not given to 2 decimal places unless already penalized in part (a).
Accept 227 515.93 from use of unrounded value.

[4 marks]
Total [7 marks]

2. (a) use of Pythagoras in 3 dimensions ***(M1)***

$AB^2 = 2^2 + 3^2 + 1^2$

$AB = 3.74 \quad (= 3.74165... = \sqrt{14})$ ***A1***

[2 marks]

(b) **METHOD 1 (using Pythagoras in 3D)**

$1^2 + 6^2 + (k-1)^2 = 14 + 1^2 + 3^2 + (k-2)^2$ ***(M1)A1***

Note: Award ***M1*** for use of Pythagoras in three dimensions, ***A1*** for correct use.

$38 - 2k = 28 - 4k$ ***(M1)***

Note: Award ***M1*** for reducing equation to linear form.

$k = -5$ ***A1***

METHOD 2 (using vectors)

find the vectors $\vec{AB}$ and $\vec{CB}$ ***(M1)***

$\vec{AB} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \quad \vec{CB} = \begin{pmatrix} -1 \\ 3 \\ k-2 \end{pmatrix}$

$\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ k-2 \end{pmatrix} = 0$ ***A1***

calculate scalar product ***(M1)***

$-2 + 9 + k - 2 = 0$

$k = -5$ ***A1***

[4 marks]

(c) **EITHER**

$\cos(B\hat{A}C) = \frac{\sqrt{14}}{\sqrt{73}}$ ***(M1)(A1)***

OR

$\cos\left(B\hat{A}C\right) = \frac{\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 6 \\ -6 \end{pmatrix}}{\sqrt{14}\sqrt{73}}$ ***(M1)(A1)***

OR

$\sin(B\hat{A}C) = \frac{\sqrt{59}}{\sqrt{73}}$ ***(M1)(A1)***

continued…

Question 2 continued

OR

$$\tan(\hat{BAC}) = \frac{\sqrt{59}}{\sqrt{14}} \qquad \textit{(M1)(A1)}$$

Note: Award ***M1*** for use of trig ratio, ***A1*** for correct use.

THEN

$$\hat{BAC} = 64.0° \quad (64.0282...^{\circ}) \quad (1.12\,(1.11750...)) \qquad \textit{A1}$$

Note: Accept 64 for the final ***A1***.

[3 marks]
Total [9 marks]

3. (a) 4 ***A1***
[1 mark]

(b)

	A	B	C	D	E	F	G	H
Brett	1	4	2	3	5	6	7	8
Clarence	1	2	4	3	5	7	6	8

(A1)(A1)

Note: Award **(A*1)*** for each line in table

$0.881\ (0.880952\ldots)$ ***A2***

Note: The ranks maybe seen within printed table.
Award ***A1*** for 0.88 (2sf) answer.

For a common mistake:
Brett: 1 3 4 2 5 6 7 8 & Clarence correct
Award (***A0)(A1)A2*****FT** if 0.952 seen.

Note: The use of $r_s = 1 - \dfrac{6\sum d^2}{n(n-1)}$ i.e. $(r_s =)1 - \dfrac{6\left(0^2+2^2+2^2+0^2+0^2+1^2+1^2+0^2\right)}{8\left(8^2-1\right)}$ may be seen which is not in the syllabus, but is a valid method because there are no ties. As they need the table to get the differences, there is no change to MS for this method.

[4 marks]

(c) (since r_s is close to 1) there is a strong (positive) correlation between the **rankings** of the sheepdogs by Brett and the **rankings** of the sheepdogs by Clarence
(since r_s is close to 1) there is a strong (positive) association between Brett's **rankings** and Clarence's **rankings**.
since r_s is not equal to 1, Brett and Clarence do not always agree on the **rankings** of the sheepdogs. ***A1***

Note: For ***A1*** to be awarded, a numerical value must be seen in part (b), and the conclusion consistent with that numerical value.
"Strong positive" is insufficient because it is not in terms of the **ranks**.
"Similar" in isolation is insufficient, however if they state "the order in which the sheepdogs were **ranked** by the judges is similar" would be awarded ***A1***.
The names of judges do not need to be seen for the ***A*** mark.

[1 mark]
Total [6 marks]

4. (a) (i) $(8,\ 4)$ ***A1***

Note: Do not condone the omission of parenthesis.

(ii) 5 ***A1***

[2 marks]

(b) (i) 8 ***A1***

(ii) **EITHER**

attempt to use Pythagoras theorem ***(M1)***

$4+(b-5)^2=9+(9-b)^2$ ***(A1)***

$(b=)\dfrac{61}{8}=7.625\quad\left(\dfrac{61}{8},7\dfrac{5}{8}\right)$ ***A2***

Note: Award ***A1A0*** for 7.63

OR

$m(BD)=-\dfrac{4}{5}$ ***(A1)***

midpoint $[BD]=(7.5,\ 7)$ ***(A1)***

Note: Gradient and midpoint may be implied by further working. Condone as interim coordinate $7.5, 7$ instead of $(7.5, 7)$.

$(y-7)=\dfrac{5}{4}(x-7.5)$

substitute $x=8$ in their perpendicular bisector ***(M1)***

$b-7=\dfrac{5}{4}(8-7.5)$

$(b=)7.625\quad\left(\dfrac{61}{8},7\dfrac{5}{8}\right)$ ***A1***

Note: Award ***A1A1M1A0*** final answer of 7.63

OR

$m(\mathrm{DF})=-4$ ***(A1)***

midpoint $[\mathrm{DF}]=(5.5\ ,7)$ ***(A1)***

Note: Gradient and midpoint may be implied by further working. Condone as interim coordinate $5.5, 7$ instead of $(5.5, 7)$

continued…

Question 4 continued

$(y-7)=\frac{1}{4}(x-5.5)$

substitute $x=8$ in their perpendicular bisector ***(M1)***

$(8-7)=\frac{1}{4}(b-5.5)$

$(b=)\,7.625\ \left(\frac{61}{8}, 7\frac{5}{8}\right)$ ***A1***

Note: Award **A*1A1M1A0*** final answer of 7.63

Note: Both a and b can be found by locating the intersection of two perpendicular bisectors. This is an inefficient method, but correct answers should be awarded full marks.

[5 marks]

Total [7 marks]

5. use of $\text{speed} = \frac{\text{distance}}{\text{time}}$ **OR** $\text{speed} \times \text{time} = \text{distance}$ **OR** $\text{time} = \frac{\text{distance}}{\text{speed}}$

seen anywhere ***(M1)***

attempt to find a bounding value of t (**OR** use appropriate values of t that support and contradict the statement that the car is speeding) ***(M1)***

$(t =) 1.25(\text{s})$ OR $(t =) 1.15(\text{s})$ ***(A1)***

THEN

EITHER

use of their upper bound for t to find minimum speed (**OR** of any value of t where $1.20481... < t \leq 1.25$ to find speed)

(speed =) $\frac{10}{1.25}$ (or $\frac{10}{\text{their time}}$) ***A1***

(speed =) $8\left(\text{m s}^{-1}\right)$ (or a value where $8 \leq \text{speed} < 8.3$) seen

hence speed $< 8.3\left(\text{ms}^{-1}\right)$ ***A1***

OR

use of their upper bound for t to find maximum distance (**OR** of any value of t where $1.20481... < t \leq 1.25$ to find distance)

(distance=) 8.3×1.25 (or $8.3 \times \text{their time}$) ***A1***

(distance =) 10.375 m (or a value where $10 < \text{distance} \leq 10.375$) seen

hence distance $> 10\left(\text{m}\right)$ ***A1***

OR

time taken at speed limit $(\text{time} =) \frac{10}{8.3}$ ***A1***

$(\text{time} =) 1.20481...(\text{s})$

hence time of $1.2(\text{s})$ to the nearest $0.1(\text{s})$ could be greater than or less than this ***A1***

THEN

conclusion: it is not certain that the car was exceeding the speed limit. ***A1***

Note: The conclusion must have been justified by further showing that for some appropriate value of t the car is speeding

Award at most ***M1M1A0A1A0A1*** for the use of a value of t where $t > 1.25$

Award at most ***M1M0A0A0A0A1*** for $t = 1.2$ used with a corresponding justified conclusion.

Total [6 marks]

6. (a) attempt to use formula $S_{n-1}^2 = \frac{n}{n-1}S_n^2$ **(*M1*)**

$= \frac{5}{4} \times 3.2^2$

$= 12.8 \left(^\circ\text{C}^2\right)$ ***A1***

[2 marks]

(b) indication of use of t-interval **(*M1*)**

$[16.1267,\ 24.0733]$ from use of 3.2

from GDC $[15.7,\ 24.5]$ $([15.6577...,\ 24.5423...])$ ***A1***

Note: Do not penalize use of round brackets, strong inequalities or using terms such as "upper" and "lower" to describe the endpoints of the interval.

[2 marks]

(c) 17 is within the confidence interval, so it is a plausible mean ***R1***

[1 mark]

Total [5 marks]

7. (a) a valid/coherent attempt to use the substitution method **(*M1*)**

$u = (-)x^2$

$\frac{\mathrm{d}u}{\mathrm{d}x} = (-)2x$ **(*A1*)**

Note: To award ***M1***, both $u =$ and $\frac{\mathrm{d}u}{\mathrm{d}x} =$ must be seen.

integral becomes

$\frac{1}{2}\int e^{-(u)}\,\mathrm{d}u$ OR $-\frac{1}{2}\int e^{u}\,\mathrm{d}u$ **(*M1*)**

$-\frac{1}{2}e^{-x^2} + c$ ***A1***

Note: Award ***M1A1M1A0*** for a final answer of $-\frac{1}{2}e^{-x^2}$ with c omitted.

Accept integration by inspection, e.g. using $\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{-x^2}\right) = -2xe^{-x^2}$

[4 marks]

(b) recognition that 0 is the lower limit ***(A1)***

substituting both 0 and k into previous expression **(*M1*)**

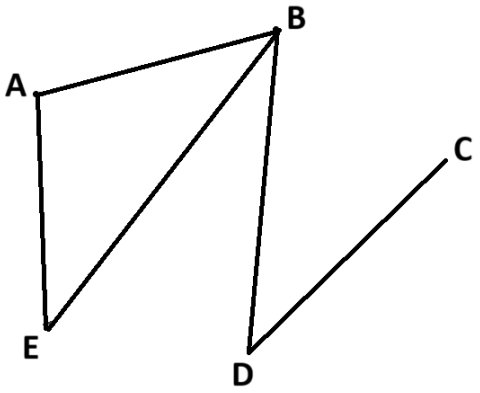
$\int_0^k xe^{-x^2}\,\mathrm{d}x = \left[-\frac{1}{2}e^{-x^2}\right]_0^k$

$= \frac{1}{2} - \frac{1}{2}e^{-k^2}$ ***A1***

[3 marks]

Total [7 marks]

8. (a)



A2

[2 marks]

Note: For graphs with edges that cross each other but are otherwise correct, award ***A1***. Award ***A2*** for a directed graph where each edge is bi-directional.

(b) recognition that A^4 is required **(*M1*)**

$$A^4 = \begin{pmatrix} 7 & 6 & 1 & 5 & 6 \\ 6 & 12 & 4 & 2 & 6 \\ 1 & 4 & 2 & 0 & 1 \\ 5 & 2 & 0 & 6 & 5 \\ 6 & 6 & 1 & 5 & 7 \end{pmatrix}$$ **(*A1*)**

recognition that diagonal entries are ones which return to the same town. **(*M1*)**
total of 34 walks ***A1***

Note: For incomplete attempts to list all possible walks of length four award ***M0A0M0A0***.

[4 marks]
Total [6 marks]

9. (a) **METHOD 1**

$A = xy$ ***A1***

attempt to use product rule ***M1***

$\frac{\mathrm{d}A}{\mathrm{d}t} = x\frac{\mathrm{d}y}{\mathrm{d}t} + y\frac{\mathrm{d}x}{\mathrm{d}t}$

$75\times(-5) + 40\times 10$

$= 25$ (km^2/year) ***A1***

METHOD 2

$\frac{\mathrm{d}x}{\mathrm{d}t} = 10, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = -5$

$A = (10t + 75)(40 - 5t)$ ***A1***

attempt to differentiate ***M1***

$\frac{\mathrm{d}A}{\mathrm{d}t} = 10(40 - 5t) - 5(10t + 75)$ OR $\frac{\mathrm{d}A}{\mathrm{d}t} = -100t + 25$

$\left(\text{at } t = 0, \ \frac{\mathrm{d}A}{\mathrm{d}t}\right) = 25$ (km^2/year) ***A1***

[3 marks]

(b) increasing since the sign of $\frac{\mathrm{d}A}{\mathrm{d}t}$ is positive ***R1***

[1 mark]

(c) new x is 85 and new y is 35, therefore new A is 2975 ***A1***

old A was 3000 ***A1***

therefore (a decrease of) $25\ (\mathrm{km}^2)$ ***A1***

Note: Accept an answer of $-25\ (\mathrm{km}^2)$

[3 marks]

Total [7 marks]

10. (a) 3

A1

[1 mark]

(b) 1

A1

[1 mark]

(c)

x	0	1	2	3	4
$f^{-1}(x)$	2	1	4	0	3

award ***A1*** for 1 correct, ***A2*** for 3 correct, ***A3*** for all 5 correct ***A1A1A1***

[3 marks]
Total [5 marks]

11. (a) $0,\ 1,\ 3$ ***A2***

Note: Award ***A1*** if either 1 or 3 is seen. Award ***A1*** for 0, 1000, 3000

[2 marks]

(b) sketching a graph or using calculus (***M1***)

attempting to find the y coordinate at the local maximum (***M1***)

local max occurs at $(0.451416...,\ 0.631130...)$ (***A1***)

$w > 0.631$ ***A1***

[4 marks]
Total [6 marks]

12. (a) $\dfrac{\mathrm{d}v}{\mathrm{d}t} = kv^{1.5}$ ***A1***

[1 mark]

(b) using initial condition to find an equation in k ***M1***

$-3 = k \times 8$

$k = -\dfrac{3}{8}\ \left(-0.375\right)$ ***A1***

attempt to separate variables ***M1***

$\int v^{-1.5}\,\mathrm{d}v = \int -\dfrac{3}{8}\mathrm{d}t$

$2v^{-0.5} = \dfrac{3}{8}t + c \quad \left(2v^{-0.5} = 0.375t + c\right)$ ***A1A1***

Note: Allow negative signs on each side. Award ***A1*** for each integral

using initial conditions to find constant of integration ***M1***

$c = 1$ ***A1***

Note: Might be a different value of c depending on rearrangement.

$v = \dfrac{1}{\left(\dfrac{3}{16}t + \dfrac{1}{2}\right)^2} \quad \left(= \dfrac{256}{\left(3t+8\right)^2}\right) \quad \left(= \dfrac{1}{\left(0.1875\,t + 0.5\right)^2}\right)$ OE ***A1***

[8 marks]
Total [9 marks]

13. let X be the number of worms in 1 square metre then $X \sim \text{Po}(1.2)$

(a) $\text{P}(X = 1) = 0.361 \quad (0.361433...)$ ***A1***

[1 mark]

(b) $\text{P}(X \geq 1) = 0.699 \quad (0.698805...)$ ***A1***

[1 mark]

(c) if Y is the number of worms in 5 square metres then $Y \sim \text{Po}(6)$ ***(A1)***

$\text{P}(Y = 5) = 0.161 \quad (0.160623...)$ ***A1***

[2 marks]

(d) **EITHER**

attempting to find an expression in terms of $\text{P}(X = 1)$ ***(M1)***

$\text{P}(X = 1)^5 = 0.00617 \quad (0.00616792...)$ ***A1***

OR

let V be the number of square metres containing exactly one worm
attempting to use the binomial with $\text{P}(X = 1)$ ***(M1)***

then $V \sim B\left(5, \text{P}\left(X = 1\right)\right)$

$\text{P}\left(V = 5\right) = 0.00617 \quad \left(0.00616792...\right)$ ***A1***

Note: Accept 0.00613 from 3sf **FT** from part (a).

[2 marks]

(e) let W be the number of square metres containing at least one worm.
attempting to use binomial ***(M1)***
then $W \sim \text{B}(5,\ \text{P}(X \geq 1))$ ***(A1)***

$\text{P}(W = 3) = 0.310 \quad (0.309572...)$ ***A1***

Note: Accept 0.31 for final ***A1.*** Accept 0.309 from 3sf **FT** from part (b).

[3 marks]

Total [9 marks]

14. use the second line since it has a higher R^2 ***A1R1***

Note: Do not award ***R0A1.*** The ***R1*** can be awarded independently within the question.

$\ln w = 0.302 \ln d + 0.693$ ***A1***

attempt to exponentiate both sides ***M1***

$w = e^{0.693} \times d^{0.302}$ ***A1***

$w = 2.00 d^{0.302} \qquad \left(= 1.99970... d^{0.302}\right)$ ***A1***

Note: Accept $w = 2d^{0.302}$ for final ***A1.***
Award ***A0R0A1M1A1A0*** for use of the first line

Total [6 marks]

15. (a) attempt to find gradient **(*M1*)**

$\frac{dy}{dx} = 2x$

gradient of tangent is 4

gradient of normal is $-\frac{1}{4}$ **(*A1*)**

equation of normal is $y - 4 = -\frac{1}{4}(x-2)$ $\left(y = -\frac{1}{4}x + \frac{9}{2}\right)$ OE ***A1***

[3 marks]

(b) attempt to find gradient ***M1***

gradient is $\frac{10-4}{0-2} = -3$ ***A1***

[2 marks]

(c) **METHOD 1**

$\theta_1 = \arctan(-3)$ $\left(\approx -1.249, -71.57^\circ\right)$ ***A1***

$\theta_2 = \arctan(-\frac{1}{4})$ $\left(\approx -0.2449, -14.04^\circ\right)$ ***A1***

Note: Award ***A1A1*** for $\arctan(3) - \arctan\left(\frac{1}{4}\right)$ OR $\arctan(-3) - \arctan\left(-\frac{1}{4}\right)$ seen

OR $\arctan\left(\frac{-3-\left(-\frac{1}{4}\right)}{1+(-3)\times\left(-\frac{1}{4}\right)}\right)$ OE.

difference between them is $1.00\ (1.00407\ldots)$ radians (57.5°) $(57.5288...^\circ)$ ***A1***

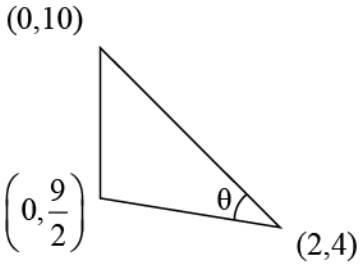
Note: Accept an obtuse angle between the normal and the ray. $2.14\ (2.13752...)$ radians, $122^\circ\ (122.471...^\circ)$.

continued...

Question 15 continued

METHOD 2

a point on the normal is $\left(0, \frac{9}{2}\right)$ ***A1***



$\cos\theta = \frac{2^2 + 6^2 + 2^2 + (0.5)^2 - (5.5)^2}{2\sqrt{4^2 + 1^2}\sqrt{1^2 + 3^2}}$ ***A1***

$\theta = 1.00\ (1.00407\ldots)\text{radians}\ \ (57.5^\circ)\ \ (57.5288...^\circ)$ ***A1***

Note: Other points on the normal such as $(18,\ 0)$ may be seen

Note: Accept an obtuse angle between the normal and the ray. $2.14\,(2.13752...)$ radians, $122^\circ\,(122.471...^\circ)$.

METHOD 3

writing the ray and normal as vectors

$\begin{pmatrix}2\\4\end{pmatrix} + \lambda\begin{pmatrix}-4\\1\end{pmatrix}$ and $\begin{pmatrix}2\\4\end{pmatrix} + \mu\begin{pmatrix}-1\\3\end{pmatrix}$ ***A1***

the angle between them is given by

$\cos\theta = \frac{\begin{pmatrix}-4\\1\end{pmatrix}\cdot\begin{pmatrix}-1\\3\end{pmatrix}}{\sqrt{(-4)^2 + 1^2}\sqrt{(-1)^2 + 3^2}}$ ***A1***

$\theta = 1.00\ (1.00407\ldots)\text{radians}\ \ (57.5^\circ)\ \ (57.5288...^\circ)$ ***A1***

Note: Accept an obtuse angle between the normal and the ray. $2.14\,(2.13752...)$ radians, $122^\circ\,(122.471...^\circ)$.

[3 marks]
Total [8 marks]

16. (a) an attempt to solve the equation for $t = 1$ showing at least one of the solutions $z = -1 + i\sqrt{7}$ or $z = -1 - i\sqrt{7}$ ***M1***

required solution $= -1 + 2.65i \quad (= -1 + 2.64575...i) \quad (= -1 + i\sqrt{7})$ ***A1***

[2 marks]

(b) using the quadratic formula ***M1***

$z = -t \pm \sqrt{t^2 - 8t}$

$= -t \pm i\sqrt{8t - t^2}$ ***A1***

attempt to find $|z|^2$ ***M1***

$|z|^2 = t^2 + 8t - t^2 = 8t$ ***A1***

this is maximised when $t = 2$ ***A1***

[5 marks]

Total [7 marks]