Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2225 – 7311

19 pages

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

Give answers to this question correct to two decimal places.

Pierre invests 1500 euros (EUR) at the end of each month for 10 years into a savings plan that pays a nominal annual interest rate of 3.6% compounded monthly.

(a) Calculate the value of Pierre's savings plan at the end of the 10 years. [3]

At the end of the 10 years, Pierre withdraws $100\,000\,\text{EUR}$ from the savings plan to use as a deposit on a house.

Pierre invests the remainder into another account for 15 years at a nominal annual interest rate of 4.5% compounded quarterly.

(b) Calculate the amount in Pierre's account at the end of this time. [4]

(This question continues on the following page)

(Question 1 continued)

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2. [Maximum mark: 9]

The point A has coordinates $(1, 2, 1)$ and the point B has coordinates $(3, 5, 2)$.

(a) Find AB. [2]

Triangle ABC is right-angled with its right angle at B. The point C has coordinates $(2, 8, k)$.

(b) Find the value of k. [4]

(c) Calculate the size of $\mathrm{B\hat{A}C}$. [3]

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3. [Maximum mark: 6]

Two judges, Brett and Clarence, rank the skill levels of eight sheepdogs in a competition. The sheepdogs are labelled A to H and the judges rank the dogs as shown in the table.

Rank	1	2	3	4	5	6	7	8
Brett	A	C	D	B	E	F	G	H
Clarence	A	B	D	C	E	G	F	H

(a) Write down the rank that Brett awards sheepdog B. [1]

(b) Calculate Spearman's rank correlation coefficient for these data. [4]

(c) Comment on your answer to part (b) in terms of the ranks awarded by Brett and Clarence. [1]

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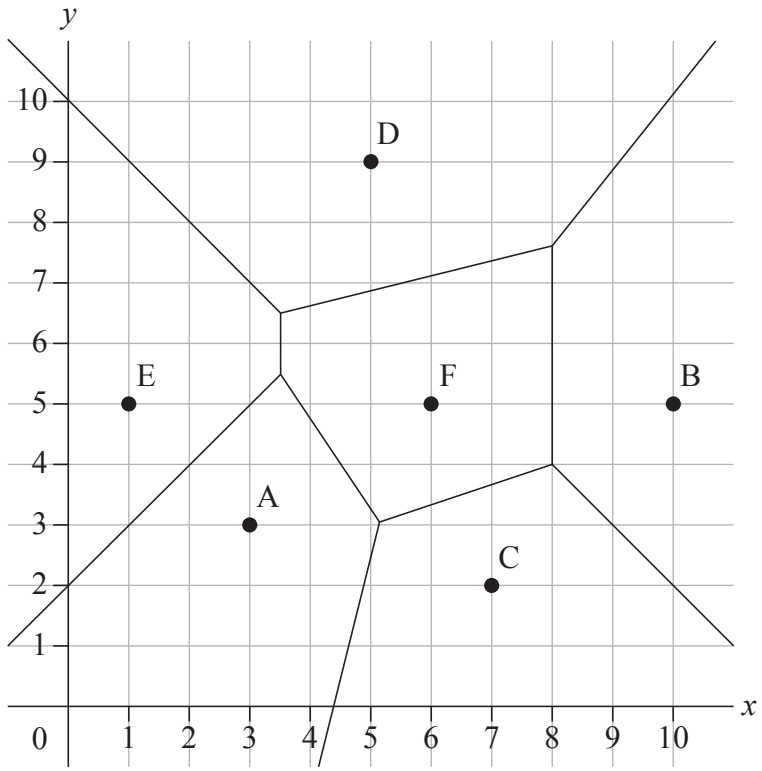
4. [Maximum mark: 7]

Consider the Voronoi diagram which shows the sites $A(3, 3)$, $B(10, 5)$, $C(7, 2)$, $D(5, 9)$, $E(1, 5)$ and $F(6, 5)$. The diagram also shows the cells formed by each site and their boundaries.



Vertex X is equidistant from sites B, C and F.

(a) (i) Write down the coordinates of X.

(ii) The exact value of BX is $\sqrt{n}$. Write down the value of n. [2]

Vertex $Y(a, b)$ is equidistant from sites B, D and F.

(b) (i) Write down the value of a.

(ii) Find the exact value of b. [5]

(This question continues on the following page)

(Question 4 continued)

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5. [Maximum mark: 6]

A speed camera is used to determine whether a car is exceeding a speed limit of $8.3\,\mathrm{m\,s^{-1}}$.

An exact distance of $10\,\mathrm{m}$ is marked out.

The car travels this $10\,\mathrm{m}$ distance in 1.2 seconds, measured to the nearest 0.1 second.

Determine whether it is certain that the car was exceeding the speed limit of $8.3\,\mathrm{m\,s^{-1}}$.

Justify your answer.

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6. [Maximum mark: 5]

The temperatures at 5 different locations on a coral reef were measured. The mean of this sample was $20.1°\text{C}$ and the standard deviation of this sample, s_n, was $3.2°\text{C}$.

(a) Find an unbiased estimate of the population variance. [2]

(b) Assuming that the temperatures are normally distributed, find a 95% confidence interval for the mean temperature on the coral reef. [2]

(c) Using your answer to part (b), determine if it is plausible that the mean temperature on the coral reef could be $17°\text{C}$. [1]

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7. [Maximum mark: 7]

(a) Find the indefinite integral $\int xe^{-x^2}dx$. [4]

(b) Hence find the area bounded by the x-axis, the curve $y = xe^{-x^2}$ and the line $x = k$. Give your answer in terms of k. [3]

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8. [Maximum mark: 6]

A mapping system stores the connections between 5 towns, labelled A, B, C, D and E, in an adjacency matrix. The adjacency matrix, with rows and columns in alphabetical order, is

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

(a) Draw and label a graph to represent the adjacency matrix. [2]

(b) Determine the number of walks of length 4 which start and end at the same town. [4]

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9. [Maximum mark: 7]

A climate scientist is modelling an ice sheet as a rectangle.

She believes that the width $(x\,\text{km})$ is increasing at a constant rate of $10\,\text{km}$ per year and the length $(y\,\text{km})$ is decreasing at a constant rate of $5\,\text{km}$ per year.

The time, t, is measured in years, and the area, A, is measured in km^2.

When $t = 0$ then $x = 75$ and $y = 40$.

(a) Find $\frac{\mathrm{d}A}{\mathrm{d}t}$ when $t = 0$. [3]

(b) State, with justification, whether the area of the ice sheet is increasing or decreasing when $t = 0$. [1]

(c) Find the change in the area of the ice sheet between $t = 0$ and $t = 1$. [3]

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10. [Maximum mark: 5]

Consider the following function, $f(x)$, defined on the domain of integers from 0 to 4 inclusive.

x	0	1	2	3	4
$f(x)$	3	1	0	4	2

(a) Solve $f(x) = 4$. [1]

(b) Solve $f(x) = x$. [1]

(c) Complete the following table.

x	0	1	2	3	4
$f^{-1}(x)$					

[3]

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11. [Maximum mark: 6]

A biologist believes that there is a relationship between the possible population size of a group of birds (p thousand) and the population of a colony of wasps (w thousand). Based on her research she believes that the relationship is

$$w = p^3 - 4p^2 + 3p.$$

(a) When $w = 0$, find the possible values of p. [2]

(b) Determine the positive values of w for which there is only one positive value of p. [4]

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12. [Maximum mark: 9]

An engineer's model for an object's motion is that its acceleration, $\frac{\mathrm{d}v}{\mathrm{d}t}$, is proportional to $v^{1.5}$, where v is its velocity measured in $\mathrm{m\,s^{-1}}$.

(a) Write down a differential equation based on the engineer's belief. [1]

The initial velocity of the object is $4\,\mathrm{m\,s^{-1}}$ and its initial acceleration is $-3\,\mathrm{m\,s^{-2}}$.

(b) Use the engineer's model to find an expression for the velocity of the object after t seconds. [8]

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13. [Maximum mark: 9]

A biologist uses a wire frame to count the number of worms in a $1\,\text{m}^2$ section.

She models the number of worms found in each $1\,\text{m}^2$ section as following a Poisson distribution with mean 1.2.

(a) Find the probability of observing exactly one worm in one $1\,\text{m}^2$ section. [1]

(b) Find the probability of observing at least one worm in one $1\,\text{m}^2$ section. [1]

The biologist looks at 5 independent $1\,\text{m}^2$ sections.

(c) Find the probability of observing a total of five worms in 5 sections. [2]

(d) Find the probability of observing exactly one worm in all 5 sections. [2]

(e) Find the probability of observing at least one worm in exactly 3 of the 5 sections. [3]

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14. [Maximum mark: 6]

A vet wants to find a relationship between the age in days of a breed of puppy (d) and its weight in $\text{kg}(w)$.

To do this he collects a large quantity of data and plots two graphs.

He finds the regression line for each graph. His results are summarized in the table.

	Horizontal axis	Vertical axis	Gradient	Intercept on vertical axis	R^2
Graph 1	d	$\ln w$	0.00571	1.54	0.72
Graph 2	$\ln d$	$\ln w$	0.302	0.693	0.95

Based on these results, find the best of the two possible relationships between w and d.

Express your relationship in the form $w = f(d)$ where f is a simplified expression.

Justify your choice of expression.

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15. [Maximum mark: 8]

An astronomer models the shape of a parabolic mirror using the equation $y = x^2$.

(a) Find the equation of the normal to the mirror at the point $(2, 4)$. [3]

A ray of light comes from an object at coordinates $(0, 10)$ and hits the mirror at the point $(2, 4)$.

(b) Find the gradient of the ray of light. [2]

(c) Find the angle between the ray of light and the normal to the mirror. [3]

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16. [Maximum mark: 7]

An electrical engineer models a circuit using the equation

$$z^2 + 2tz + 8t = 0$$

where t is the time in seconds and $0 \leq t \leq 2$.

(a) When $t = 1$, find the value of z which satisfies $\frac{\pi}{2} < \arg z < \pi$. Give your answer in the form $a + bi$. [2]

The power in the circuit is given by $|z|^2$.

(b) Find the value of t in the interval $0 \leq t \leq 2$ for which the power is maximized. [5]

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Answers written on this page
will not be marked.