Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

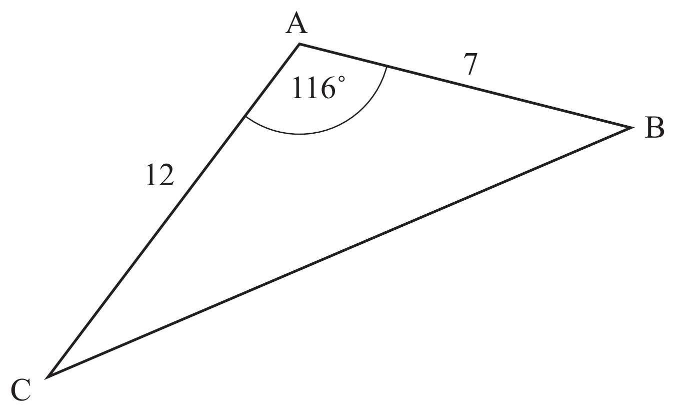
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a triangle ABC, with $\text{AB} = 7$, $\text{AC} = 12$ and $\text{B}\hat{\text{A}}\text{C} = 116°$.

diagram not to scale



(a) Find BC. [3]

(b) Find $\text{A}\hat{\text{C}}\text{B}$. [3]

2. [Maximum mark: 5]

Consider the function $f(x) = 2x^4 - 6x^3 + px^2 + qx - 2$, where p, $q \in \mathbb{R}$.
A factor of $f(x)$ is $(x - 1)$, and when $f(x)$ is divided by $(x - 3)$ the remainder is -2.

Find the value of p and of q.

3. [Maximum mark: 6]

A supermarket analyses the shopping habits of its customers.

The number of times, X, each customer visits the supermarket in a week is given by the following probability distribution.

x	1	2	3	4	5	≥ 6
$\mathrm{P}(X=x)$	$1.5a$	$2a$	0.281	a	0.026	0

(a) (i) Find the value of a.

(ii) Write down the mode of X. [3]

(b) Find the mean of X. [2]

The manager wants to know why customers come to their supermarket. They survey the first 50 customers to arrive at the supermarket on a particular day.

(c) Identify which one of the following best describes the manager's sampling method. Circle your answer. [1]

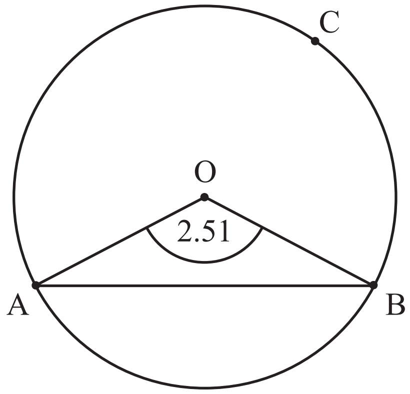
Simple random / Systematic / Convenience / Quota / Stratified

4. [Maximum mark: 5]

The following diagram shows a circle with centre O.

Points A, B and C lie on the circle.

diagram not to scale



The area of triangle AOB is $26\,\mathrm{cm}^2$ and $\mathrm{A\hat{O}B} = 2.51$ radians.

Find the length of arc ACB.

5. [Maximum mark: 7]

Consider the function $f(x)=\dfrac{(2x+a)^3}{(x+5)^2}$, where $x \neq -5$ and $a \in \mathbb{R}^+$.

(a) Find an expression for $f'(x)$, in terms of a. [3]

When $x=1$, the tangent to the graph of f makes an angle of $70°$ to the horizontal.

(b) Find the two possible values of a. [4]

6. [Maximum mark: 8]

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} -5 \\ 7 \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} -8 \\ 9 \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} p \\ -6 \end{pmatrix}$, where $p \in \mathbb{R}$.

(a) Find an expression, in terms of p, for

(i) $\boldsymbol{a} \bullet \boldsymbol{c}$;

(ii) $\boldsymbol{b} \bullet \boldsymbol{c}$. [3]

The angle between $\boldsymbol{a}$ and $\boldsymbol{c}$ is equal to the angle between $\boldsymbol{b}$ and $\boldsymbol{c}$.

(b) Find the value of p. [5]

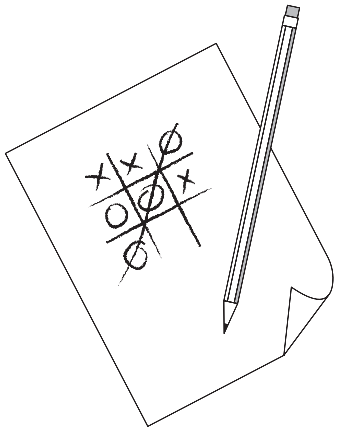
7. [Maximum mark: 6]

In the expansion of $\frac{1}{\sqrt{q-x^2}}$, where $q \in \mathbb{Q}^+$, the coefficient of x^6 is 5120.

Find the value of q.

8. [Maximum mark: 5]

A class of students plays a tic-tac-toe competition among themselves. Each individual game in the competition involves only two students.



Every student in the class is to play every other student twice. However, Stephen left the class after he had played only seven games. All other games, not involving Stephen, were played.

By the end of the competition a total of 513 games had been played.

Determine the number of students that were originally in the class.

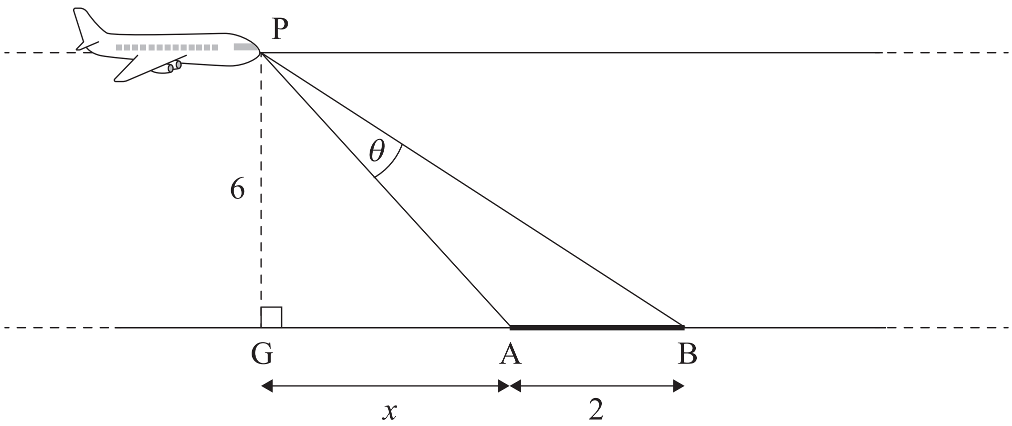
9. [Maximum mark: 9]

An airplane, P, is flying over horizontal ground at a constant height of 6 km and travelling at a constant speed. It is approaching a runway, [AB], which is 2 km in length.

Let G be the point on the ground directly below the airplane. When $\text{GA} = x\text{ km}$, the pilot's viewing angle of the runway, $\text{A}\hat{\text{P}}\text{B}$, is θ.

This is shown in the following diagram.

diagram not to scale



(a) Show that $\theta = \arctan\left(\frac{x+2}{6}\right) - \arctan\left(\frac{x}{6}\right)$. [2]

When the viewing angle is 0.178 radians, the rate at which the viewing angle is changing is 12.5 radians per hour.

(b) Find the speed of the airplane. [7]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 14]

Two athletes, Fiona and Lucy, compete in a 200 metres race along a straight track.

Fiona's velocity, in m s^{-1}, during the race can be modelled by $v(t) = \dfrac{8.14\,t}{\sqrt{t^2 + 0.2}}$, where $t \ge 0$. Time, t, is measured in seconds from when the race starts.

(a) (i) Write down the value of $v(1)$.

(ii) Find the time when Fiona's velocity is $5\,\text{m s}^{-1}$. [3]

(b) Find the time when Fiona's acceleration is $4\,\text{m s}^{-2}$. [2]

(c) (i) Write down the limit of $v(t)$ as t approaches infinity.

(ii) State a reason why the value in part (c)(i) is not valid in the context of this question. [3]

Lucy's velocity, in m s^{-1}, during the race can be modelled by $w(t) = \dfrac{8\,t}{\sqrt{t^2 + 0.3}}$, where $t \ge 0$.

Fiona completes the race and crosses the finishing line in front of Lucy.

(d) Find the distance Lucy is from the finishing line when Fiona completes the 200 metres. [6]

Do **not** write solutions on this page.

11. [Maximum mark: 18]

Amanda enters data from surveys into a database. It can be assumed that the accuracy of any survey entered is independent of all other surveys entered.

From previous records, it is known that Amanda enters 8% of the surveys inaccurately.

(a) On a particular day Amanda enters data from 50 surveys.

(i) Find the probability that Amanda entered at most six surveys inaccurately.

(ii) Given that at most six surveys were entered inaccurately, find the probability that exactly four surveys were entered inaccurately. [5]

On a different day Amanda enters data from n surveys. On this day, the probability that at most six surveys were entered inaccurately is approximately 0.367.

(b) Find the value of n. [3]

Bryce and Carmen also enter data from surveys into the same database. It is known that surveys entered by Bryce and Carmen are inaccurate 6% and 11% of the time respectively. It can again be assumed that the accuracy of any survey entered is independent of all other surveys entered.

From the surveys assigned to the three of them, Amanda enters 55%, Bryce 25% and Carmen 20%.

(c) Find the probability that a randomly selected survey was

(i) entered inaccurately;

(ii) entered by Amanda, given that the survey was entered inaccurately. [6]

The following year, the accuracy of Amanda's and Bryce's work remained the same, as did the percentage of surveys entered by each of the three employees. However, Carmen's accuracy had improved and the probability that she entered a survey inaccurately was now $x\%$.

The probability that a randomly selected survey had been entered inaccurately was now the same as the probability that Carmen made an error when entering a survey.

(d) Find the value of x. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 21]

(a) Find $\int (x^2 - 5)\,e^x\,dx$. [6]

Consider the differential equation $\dfrac{dy}{dx} = x^2 - y - 5$.

(b) By solving the differential equation, show that its solution can be expressed in the form $y = x^2 - 2x - 3 + Ce^{-x}$, where C is a constant. [4]

(c) Sketch the curve of the particular solution which passes through the point $(-3, 2)$, for $-4 \le x \le 4$, clearly labelling the coordinates of any local maximum and minimum points. [5]

Consider the family of curves that are solutions of the differential equation.

The tangent at $x = -3$ is drawn for each of these curves.

(d) By considering the curve which passes through the point $(-3, p)$ and the curve which passes through the point $(-3, q)$, where $p, q \in \mathbb{R}$, $p \neq q$, show that all these tangents intersect at a common point, and state its coordinates. [6]

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