2225 – 7107M

Markscheme

May 2025

Mathematics: analysis and approaches

Higher level

Paper 2

28 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.

- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a “show that” question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is ‘Hence’ and not ‘Hence or otherwise’ then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1,9$ or 1000 and $1,000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

Section A

1. (a) **EITHER**

$210^{\circ} = \frac{210\pi}{180}\left(= \frac{7\pi}{6} = 3.66519...\right)$ radians ***(A1)***

attempt to use radian formula for area of sector ***(M1)***

area $= \frac{1}{2}(19.5)^2\left(\frac{7\pi}{6}\right)$

OR

attempt to use degree formula for area of sector ***(M1)***

area $= \frac{210}{360}\pi(19.5)^2$ ***(A1)***

THEN

area $= \frac{3549\pi}{16} = 696.844...$

$= 697\left(= \frac{3549\pi}{16}\right)$ (cm^2) ***A1***

[3 marks]

(b) **EITHER**

arc length $= 19.5\left(\frac{7\pi}{6}\right)$ OR $= \frac{210}{360}(2\pi(19.5))\left(= \frac{91\pi}{4} = 71.4712...\right)$ ***(A1)***

attempt to set $2\pi r$ equal to arc length ***(M1)***

$2\pi r = 71.4712...$

OR

attempt to set $\pi r l$ equal to their area from (a) ***(M1)***

$19.5\pi r = 696.844...$ ***(A1)***

THEN

$r = 11.4\left(= \frac{91}{8} = 11.375\right)$ (cm) ***A1***

[3 marks]

Total[6 marks]

2. (a) period is $\frac{\pi}{2}\left(=1.57079...=1.57\right)$ ***A1***

[1 mark]

(b) attempt to substitute $x=\frac{\pi}{12}, f(x)=5$ and $x=\frac{\pi}{3}, f(x)=7$ to obtain two equations ***(M1)***

Note: accept work where x values have been converted into degrees

$a\tan\left(\frac{\pi}{6}\right)+b=5$ and $a\tan\left(\frac{2\pi}{3}\right)+b=7$ $\left(\Rightarrow \frac{a}{\sqrt{3}}+b=5 \text{ and } -a\sqrt{3}+b=7\right)$

$a=-\frac{\sqrt{3}}{2}\left(=-0.866025...=-0.866\right)$ ***A1***

$b=\frac{11}{2}\left(=5.5\right)$ ***A1***

Note: These ***A1*** marks may be awarded independently.

[3 marks]

Total [4 marks]

3. **METHOD 1**

attempt to find change in population using a definite integral (*M1)*

$t = 4$ at the start of 2026 (seen anywhere) ***(A1)***

$\int_0^4 -104000e^{-0.0145t}dt$ (***A1)***

$= -404165.8...$ (***A1)***

attempt to add initial population to their change in population from a definite integral (***M1)***

population at the start of 2026 $= 6.78 \times 10^6 - 404165.8...$

$= 6375834.1...$

$= 6380000\left(= 6.38 \times 10^6\right)$ ***A1***

METHOD 2

attempt to find population using an indefinite integral (***M1)***

$P = \int -104000e^{-0.0145t}dt$

$\frac{-104000e^{-0.0145t}}{-0.0145} + c\left(= 7172413.7...e^{-0.0145t} + c\right)$ ***(A1)***

attempt to substitute $t = 0, P = 6.78 \times 10^6$ into equation with c. ***(M1)***

$6.78 \times 10^6 = 7172413.7... + c \Rightarrow c = -392413.7...$

$P = 7172413.7...e^{-0.0145t} - 392413.7...$ ***(A1)***

$t = 4$ at the start of 2026 (seen anywhere) ***(A1)***

population at the start of 2026 $7172413.7...e^{-0.0145(4)} - 392413.7...$

$= 6375834.1...$

$= 6380000\left(= 6.38 \times 10^6\right)$ ***A1***

Total [6 marks]

4. (a) attempt to substitute $F = 19.8$ into the regression line for A on F ***(M1)***

$A = 2.89(19.8) + 99.3$

$= 156.522$ (cm)

arm span = 157 (cm) ***A1***

Note: Award ***M0A0*** for choosing the wrong regression line to get $A = 156.417...$ so $A = 156$.

[2 marks]

(b) recognition that the lines intersect at the mean point (may be seen on a sketch) ***(M1)***

$2.89F + 99.3 = \frac{F + 32.6}{0.335}$ OR $0.335A - 32.6 = \frac{A - 99.3}{2.89}$

$159.686...$ or $20.8948...$

the mean arm span $= 160$ (cm), the mean foot length $= 20.9$ (cm) ***A1A1***

[3 marks]

(c) **METHOD 1**

recognition of symmetry of interval around mean (may be seen on a sketch) ***(M1)***

$P(H < 153) = 0.06$ OR $P(H < 173) = 0.94$ OR equivalent

$\frac{153 - 163}{\sigma} = -1.55477...$ OR $\frac{173 - 163}{\sigma} = 1.55477...$ ***(A1)***

$\sigma = 6.43181...$

$\sigma = 6.43$ (cm) ***A1***

METHOD 2

attempt to find σ by equating an appropriate correct normal CDF function to 0.88 (or e.g. 0.06 or 0.94) ***(M1)***

$\sigma = 6.43181...$

$\sigma = 6.43$ (cm) ***A2***

Note: Accept use of calculator notation eg $normcdf(153, 173, 163, \sigma) = 0.88$

[3 marks]

Total [8 marks]

5. (a) recognition of the need to differentiate (***M1***)

$h'(x) = \frac{\pi}{50}\left(-15\sin\left(\frac{\pi x}{50}\right)\right)\left(= -\frac{15\pi}{50}\sin\left(\frac{\pi x}{50}\right) = -\frac{3\pi}{10}\sin\left(\frac{\pi x}{50}\right)\right)$ ***A1A1***

$h'(k) = -\frac{15\pi}{50}\sin\left(\frac{\pi k}{50}\right)\left(= -\frac{3\pi}{10}\sin\left(\frac{\pi k}{50}\right)\right)$

Note: Award ***A1*** for $-15\sin\left(\frac{\pi k}{50}\right)$ and ***A1*** for factor of $\frac{\pi}{50}$.

Award ***A1A0*** for a correct expression with additional terms or additional factors.

[3 marks]

(b) recognition that gradient of tangent = $-\tan\left(\frac{\pi}{8}\right)$ OR $\tan\left(\frac{7\pi}{8}\right)$ (***M1***)

Note: Accept $\tan\left(\frac{\pi}{8}\right)$ OR $-\tan\left(\frac{7\pi}{8}\right)$ for the (***M1***)

setting their $h'(k)$ equal to $-\tan\left(\frac{\pi}{8}\right)$ OR $\tan\left(\frac{7\pi}{8}\right)(= -0.414213...)$ (***A1***)

$-\frac{15\pi}{50}\sin\left(\frac{\pi k}{50}\right) = -\tan\left(\frac{\pi}{8}\right)$ OR $-\frac{15\pi}{50}\sin\left(\frac{\pi k}{50}\right) = \tan\left(\frac{7\pi}{8}\right)$

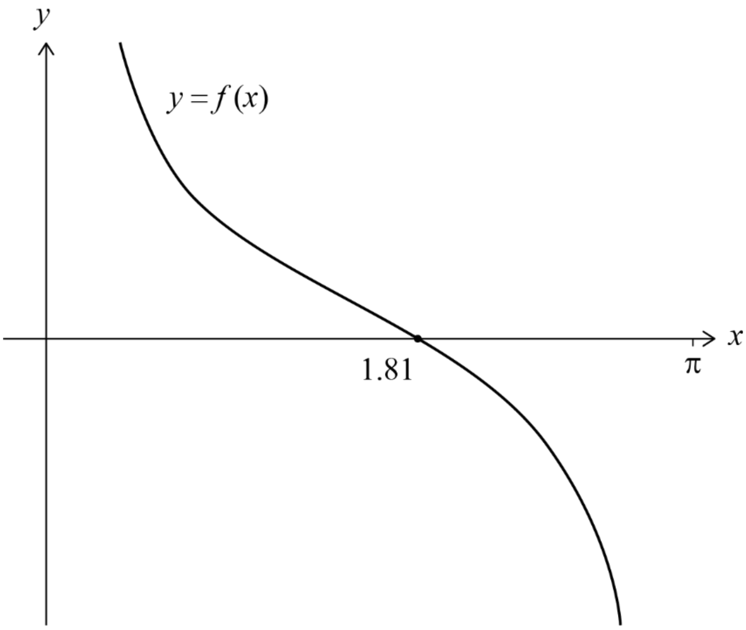
$k = 7.24211...$, $k = 42.7578...$

$k = 7.24$, $k = 42.8$ ***A1***

[3 marks]
Total [6 marks]

6. (a) (i) $\left(f(x)=\right)\dfrac{4\cos x}{\sin x}+\sin x$ ***A1***

(ii)



correct domain and curvature ***A1***

Note: The correct domain may be implied by a single branch of the function.
Condone the absence of π on the x-axis.

Award ***A0*** if the graph is not a function.

correct x-intercept at $x=1.80911...=1.81$ ***A1***

Note: Accept coordinates $(1.81, 0)$. This mark is independent of the first ***A1***.

[3 marks]

(b) $f^{-1}(2)=1.31837...$

$=1.32$ ***A1***

[1 mark]

continued…

Question 6 continued.

(c) **METHOD 1**

EITHER

recognition that $\sec x = \frac{1}{\cos x}$ ***(M1)***

$\cos\alpha = \frac{2}{3}(= 0.666666...)$

$\alpha = \arccos\frac{2}{3}(= 0.841068...)$ ***(A1)***

OR

$\alpha = \text{arcsec}\,1.5(= 0.841068...)$ ***(A2)***

THEN

$f(\alpha) = f(0.841068...) = 4.32306...$

$= 4.32\left(= \frac{29\sqrt{5}}{15}\right)$ ***A1***

METHOD 2

recognition that $\sec x = \frac{1}{\cos x}$ ***(M1)***

$\cos\alpha = \frac{2}{3}, \sin\alpha = \frac{\sqrt{5}}{3}\left(\cot x = \frac{2}{\sqrt{5}}\right)$ ***(A1)***

$f(\alpha) = \frac{4\left(\frac{2}{3}\right)}{\frac{\sqrt{5}}{3}} + \frac{\sqrt{5}}{3}\left(= 4\left(\frac{2}{\sqrt{5}}\right) + \frac{\sqrt{5}}{3}\right)$

$= \frac{29\sqrt{5}}{15}(= 4.32306... = 4.32)$ ***A1***

[3 marks]

Total [7 marks]

7. (a) recognition that the speed is the magnitude of the velocity ***(M1)***

speed $= 4\sqrt{10^2 + (-25)^2}$

$= 107.703...$

$= 108 \quad (= 20\sqrt{29})$ (km/h) ***A1***

[2 marks]

(b) **METHOD 1**

attempt to use right-angled triangle with the horizontal speed found in (a) ***(M1)***

$\tan\beta = \frac{16}{20\sqrt{29}} \left(= \frac{4}{5\sqrt{29}} \right)$ ***(A1)***

$\beta = 8.44984\left({}^\circ\right)$

$\beta = 8.45\left({}^\circ\right)$ ***A1***

Note: Award ***M1A1A0*** for answer of 0.147 radians.

METHOD 2

attempt to find angle between $\begin{pmatrix} 10 \\ -25 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 10 \\ -25 \\ 0 \end{pmatrix}$ ***(M1)***

$$\cos\beta = \frac{\begin{pmatrix} 10 \\ -25 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 10 \\ -25 \\ 0 \end{pmatrix}}{\sqrt{10^2 + (-25)^2 + (-4)^2}\sqrt{10^2 + (-25)^2}} \left(= \frac{725}{\sqrt{741}\sqrt{725}} = 0.989144... \right)$$ OR

$$\sin\beta = \frac{\left| \begin{pmatrix} 10 \\ -25 \\ -4 \end{pmatrix} \times \begin{pmatrix} 10 \\ -25 \\ 0 \end{pmatrix} \right|}{\sqrt{10^2 + (-25)^2 + (-4)^2}\sqrt{10^2 + (-25)^2}} = \frac{\left| \begin{pmatrix} -100 \\ -40 \\ 0 \end{pmatrix} \right|}{\sqrt{741}\sqrt{725}} \left(= \frac{4}{\sqrt{741}} = 0.146943... \right)$$ ***(A1)***

$\beta = 8.44984\left({}^\circ\right)$

$\beta = 8.45^\circ$ ***A1***

continued…

Question 7 continued.

METHOD 3

attempt to find angle between $\begin{pmatrix} 10 \\ -25 \\ -4 \end{pmatrix}$ and a plane parallel to $z = 0$ ***(M1)***

$$\sin\beta = \frac{\left|\begin{pmatrix} 10 \\ -25 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}\right|}{\sqrt{10^2 + (-25)^2 + (-4)^2}} \left(= \frac{4}{\sqrt{741}} = 0.146943\ldots \right)$$ ***(A1)***

Note: This could also be written as $\cos\left(90^\circ - \beta\right) = \ldots$

$\beta = 8.44984\left(^\circ\right)$

$\beta = 8.45^\circ$ ***A1***

[3 marks]

Total [5 marks]

8. (a) $\text{Im}\left(e^{2ti}\left(e^{i}+e^{3i}\right)\right)$

$= \text{Im}\left(e^{(2t+1)i}+e^{(2t+3)i}\right)$ ***A1***

Note: This ***A1*** is for clearly showing that the powers are added.
Accept alternative notation for the step of adding the arguments e.g.
$(\cos(2t)+i\sin(2t))(\cos(1)+i\sin(1)) = \cos(2t+1)+i\sin(2t+1)$

$= \sin\left(2t+1\right)+\sin\left(2t+3\right)$ ***A1***

$= h(t)$ ***AG***

Note: Accept argument in reverse

[2 marks]

(b) $e^{i}+e^{3i} = 1.08060...e^{2i}$

$1.08e^{2i}$ ***A1A1***

Note: Award ***A1*** for modulus ***A1*** for argument

$r = 1.08\left(= 2\cos(1)\right)$, $\theta = 2$

[2 marks]

(c) **METHOD 1**

attempt to use their answers to part (a) and (b) to write $h(t)$ as the imaginary part of a number in the form $re^{i\theta}$ ***(M1)***

$h(t) = \text{Im}\left(e^{2ti}\left(1.08060...e^{2i}\right)\right)$

$= \text{Im}\left(1.08060...e^{(2t+2)i}\right)$ ***(A1)***

$h(t) = 1.08060...\sin(2t+2)$

$= 1.08\sin(2t+2)\left(= 2\cos(1)\sin(2t+2)\right)$ ***A1***

METHOD 2

Considering the graph of $h(t)$

amplitude $= 1.08060...$

$p = 1.08\left(= 2\cos(1)\right)$ ***(A1)***

attempt to consider the horizontal shift of the graph of h ***(M1)***

first negative zero of graph is -1

$h(t) = 1.08060...\sin(2(t+1))\left(= 1.08060...\sin(2t+2)\right)$

$= 1.08\sin(2t+2)\left(= 2\cos(1)\sin(2t+2)\right)$ ***A1***

[3 marks]
Total [7 marks]

9. (a) attempt to use Euler's method with a step of 0.25 ***(M1)***

$\left(x_{n+1} = x_n + 0.25\right), y_{n+1} = y_n + 0.25\left(\frac{2x_n}{{x_n}^2 + y_n}\right)$

$\left(y_0 = 0\right)$
$y_1 = 0.5$
$y_2 = 0.803030...$ ***(A1)***
$y_3 = 1.04868...$

Note: Award ***(A1)*** for at least two correct intermediate values given to 3sf.

If $x = 2, y = 1.26152...$

$y = 1.26$ ***A1***

[3 marks]

(b) (i) The estimate will be an overestimate ***A1***

because the tangents to the curve lie above the curve (the curve is concave down). ***R1***

(ii) The gradient of the curve (at $(1, 0)$) is positive (so the method works in the direction of the upper part of the curve). ***R1***

[3 marks]

Total [6 marks]

Section B

10. (a) 15 ***A1***

[1 mark]

(b) attempt to add 11 cards onto a stack with 3 rows OR attempt to consider all 4 rows ***(M1)***

valid diagram with 4 rows OR $t_4 = 15 + 11$ OR $t_4 = 2 + 5 + 8 + 11$

$= 26$ ***A1***

[2 marks]

(c) **METHOD 1**

recognition that t_n is a sum of an arithmetic sequence ***(M1)***

$t_n = 2 + 5 + 8 + 11 + ...$

attempt to use formula for the sum of n terms of an arithmetic sequence ***M1***

$t_n = \frac{n}{2}\left(2(2) + 3(n-1)\right)$ ***A1***

$t_n = \frac{n}{2}(3n+1)$ ***AG***

METHOD 2

attempt to split t_n into the total number of stacked and horizontal cards ***(M1)***

stacked $2 + 4 + 6 + ... = \frac{n}{2}(4 + 2(n-1))\left(= \frac{n}{2}(2n+2)\right)$ ***A1***

horizontal $0 + 1 + 2 + ... = \frac{n}{2}(0 + 1(n-1))\left(= \frac{n}{2}(n-1)\right)$ ***A1***

$t_n = \frac{n}{2}(4 + 2(n-1)) + \frac{n}{2}(0 + 1(n-1))\left(= \frac{n}{2}(2n+2) + \frac{n}{2}(n-1)\right)$

$t_n = \frac{n}{2}(3n+1)$ ***AG***

continued…

Question 10 continued.

METHOD 3

recognition that a stack with n rows is made up of complete triangles with the bottom row of horizontal cards removed and that the numbers of complete triangle cards form an arithmetic sequence ***(M1)***

$t_n = (3+6+9+12+...+3n)-n$ OR $t_n = 3(1+2+3+4+...+n)-n$

attempt to use formula for the sum of n terms of an arithmetic sequence ***M1***

$t_n = \frac{n}{2}(2(3)+3(n-1))-n$ OR $t_n = 3\times\frac{n}{2}(1+n)-n$ ***A1***

$t_n = \frac{n}{2}(3n+1)$ ***AG***

[3 marks]

(d) attempt to solve $\frac{n(3n+1)}{2} \le 14(52)(=728)$ (***M1)***

Note: Accept an attempt to solve an equation for (***M1)***.

$21.8642...$ OR $n=21, t_n=672$ and $n=22, t_n=737$ ***(A1)***

max number of rows is 21 ***A1***

[3 marks]

continued…

Question 10 continued.

(e) **EITHER**

attempt to solve by listing at least six values of t_n (***M1***)

$2, 7, 15, 26, 40, 57...$

OR

recognition that $\dfrac{\frac{1}{2}n(3n+1)}{52}$ must be an integer ***(M1)***

$\frac{1}{2}n(3n+1) = 52k$ $\left(\text{where } k \text{ is an integer}\right)$

THEN

min number of rows is 13 ***A1***

Note: Award (***M1)A0*** for an answer of 5 packs.

Award ***M0A0*** for any answer resulting from solving $\frac{1}{2}n(3n+1) = 52$.

[2 marks]

continued…

Question 10 continued.

(f) **EITHER**

attempt to use Pythagoras's Theorem or trigonometry to find the height of an equilateral triangle with sides 88mm ***(M1)***

height= $\sqrt{88^2 - 44^2}$ OR $88\sin 60^\circ$ OR $88\cos 30^\circ$ OR $44\tan 60^\circ$ OR

$\frac{44}{\tan 30^\circ}$ OR $44\sqrt{3}\,(= 76.2102...)$ ***(A1)***

attempt to solve $44n\sqrt{3} > 2000$ OR $\text{their perpendicular height} \times n > 2000$ ***(M1)***

Note: Accept an attempt to solve an equation for (***M1)***.

OR

attempt to use trigonometry to find the side of an equilateral triangle with height 2000mm ***(M1)***

side= $\frac{2000}{\sin 60^\circ}$ OR $\frac{2000}{\cos 30^\circ}$ OR $\frac{4000}{\sqrt{3}}\,(= 2309.40...)$ ***(A1)***

attempt to solve $88n > 2309.40...$ OR $88n > \text{their side}$ ***(M1)***

Note: Accept an attempt to solve an equation for (***M1)***.

THEN

$n > 26.2431...$

so min number of rows is 27 ***(A1)***

$t_{27} = 1107$ ***A1***

[5 marks]

Total [16 marks]

11. (a) (i) $\int_{2.25}^{4.5} \frac{4}{21}\left(1-\cos\left(\frac{4\pi}{9}(t-2.25)\right)\right)\mathrm{d}t$

$= \frac{3}{7}(= 0.428571... = 0.429)$ ***A1***

(ii) mode of $T = 4.5$ ***A1***

(iii) the median is greater ***A1***

$\mathrm{P}(T < 4.5) < 0.5$ OR $\mathrm{P}(T > 4.5) > 0.5$ OR $\mathrm{P}(T < 4.5) < \mathrm{P}(T > 4.5)$ OR median $= 4.69$ ***R1***

Note: Accept reference to areas rather than probabilities.

[4 marks]

(b) recognition of the need to integrate f ***(M1)***

$\int_{2.25}^{3.5} f(t)\mathrm{d}t$

$= 0.103749\ldots$

$= 0.104$ ***A1***

[2 marks]

(c) attempt to use formula for conditional probability in context ***(M1)***

$\frac{\mathrm{P}(T \le 3)}{\mathrm{P}(\text{fast})}$ OR $\frac{\mathrm{P}(T \le 3)}{\mathrm{P}(T \le 3.5)}$ OR $\frac{\mathrm{P}(\text{very fast})}{\mathrm{P}(\text{fast})}$ (accept strict inequality signs)

$= \frac{0.0247152...}{0.103749...}$ ***(A1)***

$= 0.238220...$

$= 0.238$ ***A1***

[3 marks]

continued…

Question 11 continued.

(d) recognition that the lower quartile q is the value such that $\int_{2.25}^{q} f(t)\mathrm{d}t = 0.25$ ***(M1)***

$$\int_{2.25}^{q} \frac{4}{21}\left(1-\cos\left(\frac{4\pi}{9}(t-2.25)\right)\right)\mathrm{d}t = 0.25$$ ***(A1)***

Note: Condone the absence of $\mathrm{d}t$ for this ***A1***.

$$\left[\frac{4}{21}\left(t-\frac{9}{4\pi}\sin\left(\frac{4\pi}{9}(t-2.25)\right)\right)\right]_{2.25}^{q} = 0.25$$

$q = 4.01290...$

$q = 4.01$ ***A1***

[3 marks]

(e) attempt to find the expected value for a transformed linear variable ***(M1)***

$\mathrm{E}(P) = \mathrm{E}(a-bT) = a - b\mathrm{E}(T)$

$a - 4.723b = 100$ ***(A1)***

recognition that max score is achieved with fastest time $t = 2.25$ ***(M1)***

maximum score $a - 2.25b = 150$ ***(A1)***

$a = 195.491...,\ b = 20.2183...$

$a = 195,\ b = 20$ ***A1***

Note: these values must be given to the nearest integer for the ***A1*** to be awarded.

[5 marks]

(f) attempt to find variance for a transformed linear variable ***(M1)***

$\mathrm{Var}(P) = \mathrm{Var}(a-bT) = b^2\mathrm{Var}(T)$

$0.906(20.2183...^2) = 370.356...$

$\mathrm{Var}(P) = 370$ ***A1***

Note: accept any answer which rounds to any value between 362 and 370 inclusive based on use of less accurate values of b.

[2 marks]

Total [19 marks]

12. (a) attempt to replace x with $-x$ ***M1***

$f_n(-x) = \sum_{r=0}^{n}\left(-2(-x)^2\right)^r$ OR $-2(-x)^2 = -2x^2$ (seen anywhere) ***A1***

$f_n(-x) = f_n(x)$ ***A1***

so f_n is even for all values of n ***AG***

[3 marks]

(b) (i) $f_3(x) = 1 - 2x^2 + \left(-2x^2\right)^2 + \left(-2x^2\right)^3$ ***A1***

$= 1 - 2x^2 + 4x^4 - 8x^6$ ***AG***

(ii) $= 1 - 2x^2 + 4x^4 - 8x^6 + 16x^8$ ***A1***

[2 marks]

(c) (i) recognition of geometric series with common ratio $-2x^2$ ***(M1)***

converges for $\left|-2x^2\right| < 1 \left(\Rightarrow x^2 < \frac{1}{2}\right)$ ***(A1)***

$-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}} \left(\Rightarrow -\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2}\right)$

largest $K = \frac{1}{\sqrt{2}} \left(= \frac{\sqrt{2}}{2}\right)$ ***A1***

(ii) use of formula for S_∞ of a geometric series with first term 1, common ratio $-2x^2$ ***(M1)***

$f(x) = \frac{1}{1-\left(-2x^2\right)}$

$\left(f(x) =\right)\frac{1}{1+2x^2}$ ***A1***

$a = 1,\ b = 2$

[5 marks]

continued…

Question 12 continued.

(d) (i) g is a one-to-one function ***(A1)***

since g is a (strictly) decreasing function OR g has no points of zero gradient (turning points) ***R1***

(ii) attempt to rearrange and swop x and y (at any stage) ***(M1)***

$x = \frac{1}{1+2y^2} \Rightarrow x + 2xy^2 = 1$

$y^2 = \frac{1-x}{2x}$ ***(A1)***

$y = \pm\sqrt{\frac{1-x}{2x}}$

$g^{-1}(x) = \sqrt{\frac{1-x}{2x}}$ ***A1***

Note: Award ***A0*** if $g^{-1}(x)$ is missing.

Award **FT *A1*** marks only if f is of the correct form $f(x) = \frac{1}{a+bx^2}$

(domain) $\frac{1}{2} < x \le 1$ ***A1***

Note: This ***A1*** can be awarded independently.

[6 marks]

continued…

Question 12 continued.

(e)

Note: Throughout part (e), do not award ***A1*** marks as **FT** from part (d)

METHOD 1

curves intersect at $x = 0.5897545107...$ ***(A1)***

attempt to add the areas to the left and to the right of the point of intersection ***(M1)***

$$\int_0^{0.589...} g(x)\mathrm{d}x + \int_{0.589...}^{1} g^{-1}(x)\mathrm{d}x \left(= \int_0^{0.589...} \frac{1}{1+2x^2}\mathrm{d}x + \int_{0.589...}^{1} \sqrt{\frac{1-x}{2x}}\mathrm{d}x \right)$$

$= 0.491548... + 0.143738...$ ***(A1)***

Note: Award ***A1*** for one correct value seen, dependent on ***(M1)***

$= 0.635286...$

Area of $R = 0.635$ ***A1***

METHOD 2

curves intersect at $x = 0.5897545107...$ ***(A1)***

attempt to find the area between $y = g(x)$ and $y = x$ to the left of the point of intersection ***(M1)***

$$\int_0^{0.589...} g(x)\mathrm{d}x - \int_0^{0.589...} x\mathrm{d}x = \int_0^{0.589...} \left(\frac{1}{1+2x^2} - x \right)\mathrm{d}x$$

$= 0.317643...$ ***(A1)***

$2(0.3176432617...) = 0.635286...$

Area of $R = 0.635$ ***A1***

continued…

Question 12 continued.

METHOD 3

curves intersect at $x = 0.5897545107...$ ***A1***

attempt to find the area under $y = g^{-1}(x)$ and $y = x$ to the right of the point of intersection and the area of a square of side $x = 0.589...$ ***(M1)***

$$0.589...^2 + 2\int_{0.589...}^{1} g^{-1}(x)\,dx \left(= 0.589...^2 + 2\int_{0.589...}^{1} \sqrt{\frac{1-x}{2x}}\,dx\right)$$

$= 0.347810... + 2(0.143738...)$ ***A1***

Note: Award ***A1*** for $0.143..$ seen, dependent on ***(M1)***

$= 0.635286...$

Area of $R = 0.635$ ***A1***

[4 marks]

Total [20 marks]