Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Consider the function $f(x) = x^3 + 5x^2 - 8$, where $x \in \mathbb{R}$.

(a) Find $f'(1)$. [2]

(b) Find the equation of the tangent to the graph of f at $x = 1$. [2]

2. [Maximum mark: 5]

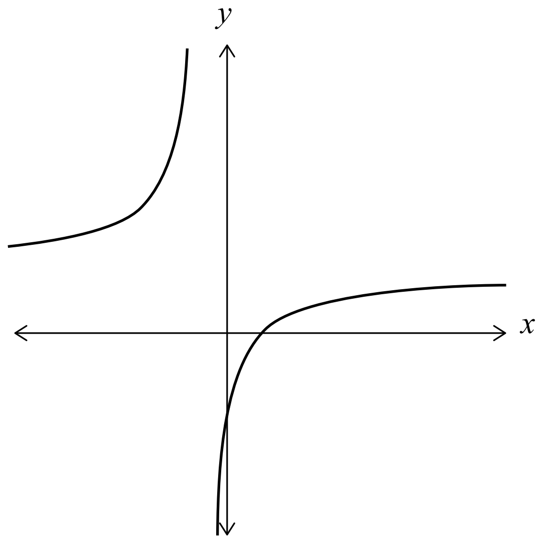
The derivative of a function g is given by $g'(x) = \cos x + e^{2x}$, where $x \in \mathbb{R}$.

Given that $g(0) = 7$, find $g(x)$.

3. [Maximum mark: 5]

The function f is defined by $f\left(x\right)=\dfrac{3x-2}{2x+1}$ for $x\in\mathbb{R}$, $x\neq-\dfrac{1}{2}$.

The following diagram shows part of the graph of $y=f(x)$.



(a) Write down the value of $f(0)$. [1]

(b) Write down the equation of the horizontal asymptote. [1]

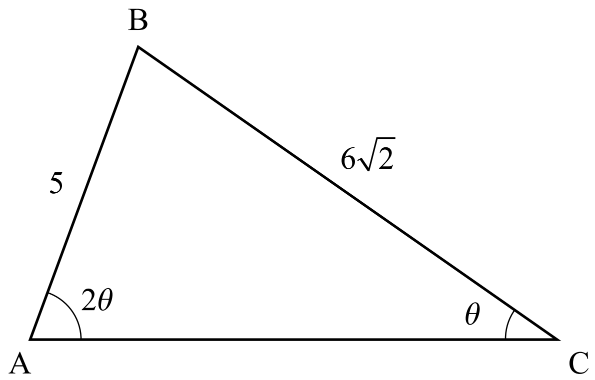
The function g is defined by $g(x)=-f(x)$ for $x\geq 0$.

(c) Find the range of g. [3]

4. [Maximum mark: 7]

The following diagram shows a non-right angled triangle ABC.

diagram not to scale



$\mathrm{AB} = 5$, $\mathrm{BC} = 6\sqrt{2}$, $\mathrm{A\hat{C}B} = \theta$ and $\mathrm{B\hat{A}C} = 2\theta$, where $0 < \theta < \frac{\pi}{2}$.

(a) Using the sine rule, show that $\cos\theta = \frac{3\sqrt{2}}{5}$. [3]

(b) Hence, find $\sin\theta$. [2]

Point D is located on $[\mathrm{AC}]$ such that the area of triangle BCD is $2\sqrt{14}$.

(c) Find DC. [2]

5. [Maximum mark: 7]

The quadratic equation $x^2 + kx + 15 - k = 0$ has two distinct real roots.

(a) Find the possible values of k. [5]

(b) Find the possible values of k in the case where the two distinct real roots are both positive or both negative. [2]

6. [Maximum mark: 7]

Consider the function $f(x)=\sqrt{x^2\ln x+4-x^2}$, where $x\in\mathbb{R}$, $x>0$.

(a) Show that the distance, l, between the origin and any point on the graph of f is given by $l=\sqrt{x^2\ln x+4}$. [1]

(b) Hence, find the x-coordinate of the point on the graph of f which is closest to the origin. [6]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

A discrete random variable, X, has the following probability distribution, where $a > 0$ and k is a constant.

x	0	a	$2a$	$3a$
$\mathrm{P}(X = x)$	k	$3k^2$	$2k^2$	k^2

(a) Show that $k = \frac{1}{3}$. [5]

(b) Find $\mathrm{P}(X < 3a)$. [2]

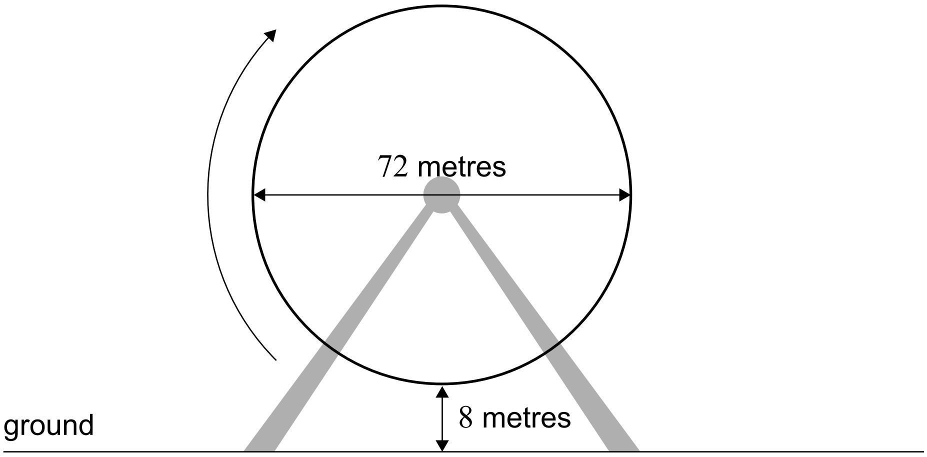
(c) Find $\mathrm{P}(X \geq a \mid X < 3a)$. [3]

(d) Given that $\mathrm{E}(X) = 20$, find the value of a. [3]

Do **not** write solutions on this page.

8. [Maximum mark: 15]

A Ferris wheel with diameter 72 metres rotates at a constant speed. The lowest point on the wheel is 8 metres above the ground, as shown on the following diagram. A seat starts at the lowest point.



The wheel completes one revolution in 12 minutes.

After t minutes, the height, h metres, of the seat above the ground is given by $h(t) = a\cos(bt) + 44$, where a, $b \in \mathbb{R}$.

(a) Find the value of

(i) a;

(ii) b. [4]

(b) Sketch the graph of h, for $0 \le t \le 12$. Clearly label the coordinates of the maximum point and the end points. [2]

Consider one revolution of the Ferris wheel.

(c) The seat is at least $26\,\text{m}$ above the ground for T minutes. Find the value of T. [5]

(d) (i) Find $h'(t)$.

(ii) Hence or otherwise, find the time at which the height of the seat above the ground is decreasing at its fastest rate. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 17]

The function f is defined by $f(x) = 4^x$, where $x \in \mathbb{R}$.

(a) Find $f^{-1}(8)$. Express your answer in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$. [3]

The function g is defined by $g(x) = 1 + \log_2 x$, where $x \in \mathbb{R}^+$.

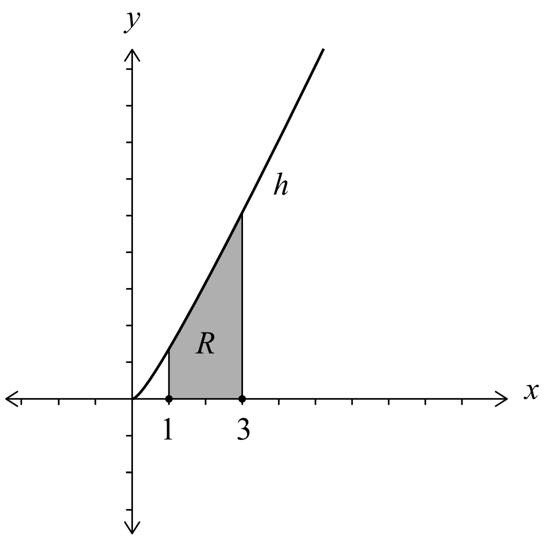
(b) (i) Find an expression for $g^{-1}(x)$.

(ii) Describe a sequence of transformations that transforms the graph of $y = g^{-1}(x)$ to the graph of $y = f(x)$. [4]

(c) Show that $(f \circ g)(x) = 4x^2$. [3]

The function h is defined by $h(x) = \frac{4x^2}{2x+1}$, $x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of h. Let R be the region enclosed by the graph of h and the x-axis, between the lines $x = 1$ and $x = 3$.



(d) (i) Show that $2x - 1 + \frac{1}{2x+1} = \frac{4x^2}{2x+1}$.

(ii) Hence or otherwise, find the area of R, giving your answer in the form $p + q\ln r$, where $p, q, r \in \mathbb{Q}^+$. [7]

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will not be marked.

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